Day 338 **A Level Maths: C Tier 10-a-day: Model Solutions**

1 Recurrence relations (AQA, Edexcel, OCR, MEI, Eduqas, CIE)

A sequence is defined by $u_{n+1} = -2u_n + 3$, $u_1 = -1$.

Find u_2, u_3 and u_4.

$u_2 = (-2) \times (-1) + 3 = 5$

$u_3 = (-2) \times 5 + 3 = -7$

$u_4 = (-2) \times (-7) + 3 = 17$

Answer: $u_2 = 5$, $u_3 = -7$, $u_4 = 17$

2 2D vectors: magnitude and direction (All boards)

The vector $\mathbf{p} = -4\mathbf{i} - 3\mathbf{j}$.

Find (a) $|\mathbf{p}|$, (b) the angle p makes with the positive x-axis, measured anticlockwise (1 d.p.).

(a) $|\mathbf{p}| = \sqrt{4^2 + 3^2} = 5$

(b) Acute angle $= \tan^{-1}(\frac{3}{4}) = 36.9°$

p is in the third quadrant, so the angle is 216.9°

Answer: (a) 5 (b) 216.9°

3 Position vector under constant velocity (AQA, Edexcel, OCR, MEI, Eduqas)

A particle moves with constant velocity $(4\mathbf{i} - 3\mathbf{j})$ m s^{-1}.

At time t = 0 its position vector is $(\mathbf{i} + 7\mathbf{j})$ m.

(a) Find its position vector when t = 2.

(b) Find the distance it travels in 2 s.

(a) $\mathbf{r} = \mathbf{r}_0 + \mathbf{v}t = (\mathbf{i} + 7\mathbf{j}) + 2(4\mathbf{i} - 3\mathbf{j})$

(b) speed $= \sqrt{4^2 + 3^2} = 5$ m s^{-1}

distance = 5 × 2

Answer: (a) (9i + j) m (b) 10 m

4 Standard deviation from raw data (AQA, Edexcel, OCR, MEI, Eduqas, CIE)

Here are 7 values: 9, 8, 20, 5, 6, 4, 10

Find the mean and the standard deviation, to 3 s.f.

$\Sigma x = 62$, $\Sigma x^2 = 722$, $n = 7$

Mean $= 62 \div 7 = 8.857$

Variance $= \frac{722}{7} - (8.857)^2 = 24.69$

Standard deviation $= \sqrt{24.69}$

Answer: mean = 8.86, standard deviation = 4.97

5 Stationary points from a sketch (diagram) (All boards)

The diagram shows the curve $y = x^3 - 3x + 5$.

A and B are turning points. Find the coordinates of A and B.

$dy/dx = 3x^2 - 3$

$dy/dx = 0$ gives $x = -1$ or $x = 1$

Substitute: $y = 7$ at $x = -1$ and $y = 3$ at $x = 1$

Answer: A (−1, 7), B (1, 3)

6 Perpendicular bisector (AQA, Edexcel, OCR, MEI, Eduqas, CIE)

A is (−5, −6) and B is (−1, −4).

Find the equation of the perpendicular bisector of AB, in the form ax + by = c with integer a, b and c.

Midpoint of AB = (−3, −5)

Gradient of AB $= \frac{2}{4} = \frac{1}{2}$, so perpendicular gradient = −2

$y - (-5) = -2(x - (-3))$

Multiply out and rearrange.

Answer: $2x + y = -11$

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7 Solve a^x = b using logarithms (All boards)

Solve $1.5^x = 10$.
Give your answer to 3 significant figures.

Take logs of both sides: $x \log 1.5 = \log 10$

$x = \frac{\log 10}{\log 1.5} = 5.68$

Answer: $x = 5.68$

8 Two-tailed binomial test (AQA, Edexcel, OCR, MEI, Eduqas, CIE)

$X \sim B(12, p)$. H_0: $p = 0.3$, H_1: $p \neq 0.3$. In a sample, $x = 1$.
Test at the 5% significance level.

Two-tailed test, so compare with $0.05 \div 2 = 0.025$
$P(X \leq 1) = 0.0850$
$0.0850 > 0.025$, so do not reject H_0

Answer: Do not reject H_0 ($P(X \leq 1) = 0.0850$)

9 Geometric sequences (AQA, Edexcel, OCR, MEI, Eduqas, CIE)

In a geometric sequence $u_3 = 36$ and $u_4 = -108$.
Find the first term and the common ratio.

$r = u_4 \div u_3 = -108 \div 36 = -3$

$a = u_3 \div r^2 = 4$

Answer: $a = 4$, $r = -3$

10 Smooth inclined plane: acceleration and speed (All boards)

A particle of mass 3 kg slides from rest down a smooth plane inclined at angle α to the horizontal, where $\sin\alpha = \frac{4}{5}$. Take $g = 9.8$ m s^{-2}.
Find (a) its acceleration, (b) its speed after 3 s.

(a) Parallel to slope: $mg \sin\alpha = ma$

$a = 9.8 \times \frac{4}{5}$

(b) $v = u + at = 0 + 7.84 \times 3$

Answer: (a) 7.84 m s^{-2} (b) 23.52 m s^{-1}