Day 336 **A Level Maths: C Tier 10-a-day: Model Solutions**

1 Intercepts of a cubic graph (AQA, Edexcel, OCR, MEI, Eduqas, CIE)

Sketch $y = 2(x + 2)(x + 1)(x - 2)$. Write down the coordinates of the points where the curve meets the axes.

$y = 0$: $x = -2, -1, 2$
$x = 0$: $y = 2 \times 1 \times (-2) \times 2 = -8$
Answer: $(-2, 0)$, $(-1, 0)$, $(2, 0)$, $(0, -8)$

2 Addition rule (AQA, Edexcel, OCR, MEI, Eduqas, CIE)

For events A and B, $P(A) = 0.3$, $P(B) = 0.3$ and $P(A \cup B) = 0.5$.
(a) Find $P(A \cap B)$.
(b) Find $P(A' \cap B')$.

(a) $P(A \cap B) = P(A) + P(B) - P(A \cup B) = 0.3 + 0.3 - 0.5 = 0.1$
(b) $A' \cap B'$ is outside both, so $P = 1 - P(A \cup B) = 1 - 0.5$
Answer: (a) 0.1 (b) 0.5

3 Stationary points of a cubic (All boards)

$y = x^3 - 3x + 3$.
Find the coordinates of the stationary points and determine their nature.

$dy/dx = 3x^2 - 3 = 3(x + 1)(x - 1) = 0$
$x = -1$: $y = 5$; $x = 1$: $y = 1$
$d^2y/dx^2 = 6x + 0$: at $x = -1$ it is -6 (maximum), at $x = 1$ it is 6 (minimum)
Answer: $(-1, 5)$ maximum; $(1, 1)$ minimum

4 Normal distribution: inverse (AQA, Edexcel, OCR, MEI, Eduqas, CIE)

$X \sim N(25, 2^2)$. Given that $P(X < a) = 0.8$, find a to 3 s.f.

$P(Z < z) = 0.8$, so $z = 0.8416$ (from the inverse normal function)
$a = 25 + 0.8416 \times 2$
Answer: 26.7

5 The factor theorem and factorising (AQA, Edexcel, OCR, MEI, Eduqas, CIE)

$f(x) = 2x^3 - 4x^2 - 38x + 40$
(a) Show that $(x + 4)$ is a factor of $f(x)$.
(b) Factorise $f(x)$ completely.

$f(-4) = (-128) + (-64) + 152 + 40 = 0$, so $(x + 4)$ is a factor.
$f(x) = (x + 4)(2x^2 - 12x + 10)$
$2x^2 - 12x + 10 = 2(x - 1)(x - 5)$
Answer: (b) $f(x) = 2(x + 4)(x - 1)(x - 5)$

6 Quadratic in sin or cos (All boards)

Solve $4\sin^2 x - 1 = 0$ for $0° \leq x \leq 360°$.

Factorise: $(2 \sin x + 1)(2 \sin x - 1) = 0$ (as a quadratic in $\sin x$)
$\sin x = -\frac{1}{2}$ or $\sin x = \frac{1}{2}$
$x = 30°, 150°, 210°, 330°$
Answer: $x = 30°, 150°, 210°, 330°$

7 Starting from rest: find the time (AQA, Edexcel, OCR, MEI, Eduqas, CIE)

A car starts from rest and moves in a straight line with constant acceleration 3 m s^{-2}.
It travels 24 m. Find the time taken.

$s = ut + \frac{1}{2}at^2$ with $u = 0$
$24 = \frac{1}{2} \times 3 \times t^2$
$t^2 = 16$
Answer: 4 s

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8 Tangent condition for a line and a curve (AQA, Edexcel, OCR, MEI, Eduqas, CIE)

The line $y = -3x + k$ is a tangent to the curve
$y = x^2 - 5x - 1$.
Find the value of k.

$x^2 - 5x - 1 = -3x + k$
$x^2 - 2x + (-1 - k) = 0$
A tangent touches once: discriminant = 0
$(-2)^2 - 4(-1 - k) = 0$
Answer: $k = -2$

9 Position vector under constant velocity (AQA, Edexcel, OCR, MEI, Eduqas)

A particle moves with constant velocity $(5\mathbf{i} + 12\mathbf{j})$ m s^{-1}.
At time t = 0 its position vector is $(4\mathbf{i} - 2\mathbf{j})$ m.
(a) Find its position vector when t = 6.
(b) Find the distance it travels in 6 s.

(a) $\mathbf{r} = \mathbf{r}_0 + \mathbf{v}t = (4\mathbf{i} - 2\mathbf{j}) + 6(5\mathbf{i} + 12\mathbf{j})$

(b) speed $= \sqrt{5^2 + 12^2} = 13$ m s^{-1}
distance $= 13 \times 6$
Answer: (a) $(34\mathbf{i} + 70\mathbf{j})$ m (b) 78 m

10 Newton-Raphson: one or two steps (AQA, Edexcel, OCR, MEI, Eduqas)

$f(x) = \ln x + x - 2$.
Use the Newton-Raphson method with $x_0 = 2$ to find x_1,
giving your answer to 3 decimal places.

$f'(x) = \frac{1}{x} + 1$

$x_{n+1} = x_n - \frac{f(x_n)}{f'(x_n)}$

$x_1 = 2 - \frac{0.6931}{1.500} = 1.5379$

Answer: $x_1 = 1.538$