**Day 334** **A Level Maths: C Tier 10-a-day: Model Solutions**

## 1 Differentiate e^x, ln x, sin and cos (All boards)

Differentiate with respect to x:
(a) 2 ln x
(b) $2\,e^{5x}$

(a) $dy/dx = \frac{2}{x}$

(b) $dy/dx = 10\,e^{5x}$

**Answer: (a) $\frac{2}{x}$ (b) $10\,e^{5x}$**

## 2 Equation of a circle (AQA, Edexcel, OCR, MEI, Eduqas, CIE)

A circle has centre $(-3, -5)$ and passes through the point $(1, -6)$. Find the equation of the circle.

$r^2 = (1 - (-3))^2 + (-6 - (-5))^2$
$= 16 + 1 = 17$
Equation: $(x - a)^2 + (y - b)^2 = r^2$
**Answer: $(x + 3)^2 + (y + 5)^2 = 17$**

## 3 Standard deviation from summary data (AQA, Edexcel, OCR, MEI, Eduqas, CIE)

For 8 values, $\Sigma x = 176$ and $\Sigma x^2 = 3936$.
Find the mean and the standard deviation.

Mean $= 176 \div 8 = 22$

Variance $= \frac{3936}{8} - 22^2 = 492 - 484 = 8$

Standard deviation $= \sqrt{8}$
**Answer: mean = 22, standard deviation = 2.83**

## 4 Parametric equations: finding points (Edexcel, OCR, MEI, CIE, AQA, Eduqas)

A curve has parametric equations
$x = 4t + 2$, $y = t^2 + 4$.
The point P has y-coordinate 8 and positive t.
Find the coordinates of P.

$t^2 + 4 = 8$, so $t^2 = 4$, giving $t = 2$ $(t > 0)$
$x = 4 \times 2 + 2 = 10$
**Answer: (10, 8)**

## 5 Resultant of perpendicular forces (All boards)

Two forces of magnitudes 12 N and 5 N act on a small ring at right angles to each other. Find (a) the magnitude of the resultant, (b) the angle it makes with the 12 N force, to 1 d.p.

(a) $R^2 = 12^2 + 5^2 = 169$
$R = \sqrt{169}$

(b) $\tan\theta = \frac{5}{12}$

**Answer: (a) 13 N (b) 22.6°**

## 6 Normal distribution: probabilities (AQA, Edexcel, OCR, MEI, Eduqas, CIE)

The times, X minutes, taken to complete a puzzle are modelled by $N(60, 6^2)$.
Find $P(X > 69)$ to 3 s.f.

$z = (69 - 60) \div 6 = 1.5$
$P(X > 69) = P(Z > 1.5)$
**Answer: 0.0668**

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## 7 Separable differential equations (All boards)

Given that dy/dx = 4xy and y = 2 when x = 0,
find y in terms of x.

$\frac{1}{y}$ dy = 4x dx, so ln y = $2x^2$ + c

x = 0, y = 2: c = ln 2

y = 2 $e^{2x^2}$

**Answer: y = 2 $e^{2x^2}$**

## 8 Velocity-time graph: acceleration from two stages (AQA, Edexcel, OCR, MEI, Eduqas, CIE)

The graph shows the velocity of a runner.
(a) Find the acceleration in the first 2 s.
(b) Find the deceleration in the last 2 s.
(c) Find the total distance travelled.

(a) a = $\frac{12 - 4}{2}$

(b) a = $\frac{12 - 0}{2}$, a deceleration of 6

(c) area = $\frac{1}{2}(4 + 12) \times 2 + 12 \times 10 + \frac{1}{2} \times 2 \times 12$

**Answer: (a) 4 m $s^{-2}$ (b) 6 m $s^{-2}$ (c) 148 m**

## 9 Solve with a shifted angle (All boards)

Solve cos(x + 30°) = $-\frac{1}{2}$ for 0° ≤ x ≤ 360°.

Let u = x + 30°, so 30° ≤ u ≤ 390°

cos u = $-\frac{1}{2}$: u = 120°, 240°

x = u − 30°: x = 90°, 210°

**Answer: x = 90°, 210°**

## 10 Quadratic inequalities (AQA, Edexcel, OCR, MEI, Eduqas, CIE)

Solve $-x - 5 < 4$.

Solve $-x^2 - 5x - 4 = 0$: $-(x + 4)(x + 1) = 0$
Critical values x = −4 and x = −1.
The graph is an ∩-shaped parabola, so take the regions outside the roots.
**Answer: x < −4 or x > −1**