Day 332 **A Level Maths: C Tier 10-a-day: Model Solutions**

1 Braking: deceleration and time (AQA, Edexcel, OCR, MEI, Eduqas, CIE)

A cyclist travelling at 15 m s^{-1} brakes uniformly and comes to rest after travelling 60 m.
(a) Find the deceleration.
(b) Find the time taken to stop.

(a) $v^2 = u^2 + 2as$: $0 = 15^2 + 2 \times a \times 60$
so $a = -1.875$, a deceleration of 1.875

(b) $s = \frac{(u + v)}{2}$ t: $60 = 7.5$ t

Answer: (a) 1.875 m s^{-2} (b) 8 s

2 Binomial: at least one success (AQA, Edexcel, OCR, MEI, Eduqas, CIE)

Each shot hits the target with probability 0.25.
Find the probability of at least one hit in 3 shots, to 3 s.f.

P(zero) = 0.75^3
P(at least one) = $1 - 0.75^3$
Answer: 0.578

3 The remainder theorem (AQA, Edexcel, OCR, MEI, Eduqas, CIE)

Find the remainder when $-3x^3 - 3x^2 - 5x + 6$ is divided by $(2x - 2)$.

Put $2x - 2 = 0$: $x = 1$
Remainder = f(1) = (−3) + (−3) + (−5) + 6
= −5
Answer: −5

4 Tree diagrams (AQA, Edexcel, OCR, MEI, Eduqas, CIE)

The tree diagram shows probabilities for events R and L.
Find the probability that exactly one of R and L happens.

Exactly one: (R and L′) or (R′ and L)
= 0.3 × 0.1 + 0.7 × 0.2
Answer: 0.17

5 Reciprocal graphs and asymptotes (AQA, Edexcel, OCR, MEI, Eduqas, CIE)

Sketch $y = \frac{5}{x - 1} + 3$.

Write down the equations of the asymptotes and the coordinates where the curve meets the axes.

Asymptotes: x = 1 and y = 3.

y = 0: $\frac{5}{x - 1} = -3$, so $x = -\frac{2}{3}$

x = 0: $y = \frac{5}{-1} + 3 = -2$

Answer: x = 1, y = 3 ; $(-\frac{2}{3}, 0)$, (0, −2)

6 Double angle equation (All boards)

Solve $\cos 2x = -\frac{1}{2}$ for $0° \leq x \leq 180°$.

Let u = 2x, so $0° \leq u \leq 360°$

$\cos u = -\frac{1}{2}$: u = 120°, 240°, 480°, 600°

Divide by 2: x = 60°, 120°
Answer: x = 60°, 120°

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7 Differentiating displacement to find velocity (AQA, Edexcel, OCR, MEI, Eduqas, CIE)

A particle moves in a straight line. Its displacement from O at time t seconds is s metres, where $s = 2t^3 + 4t^2 + 3t$.
Find its velocity when t = 4.

$v = \frac{ds}{dt} = 6t^2 + 8t + 3$

When t = 4: $v = 6 \times 4^2 + 8 \times 4 + 3$

Answer: 131 m s^{-1}

8 Rewrite a sin x + b cos x in R form (All boards)

Express 12 sin x + 5 cos x in the form R sin(x + α), where R > 0 and 0° < α < 90°.
Give α to 1 decimal place.

R sin(x + α) = R sin x cos α + R cos x sin α
R cos α = 12 and R sin α = 5

$R = \sqrt{12^2 + 5^2} = 13$, $\tan \alpha = \frac{5}{12}$, α = 22.6°

Answer: 13 sin(x + 22.6°)

9 Proof by contradiction (AQA, Edexcel, OCR, MEI, Eduqas)

Prove by contradiction that there are no integers x and y such that 4x + 6y = 97.

Assume that integers x and y exist with 4x + 6y = 97.
Then 2(2x + 3y) = 97, so 97 would be a multiple of 2.
But 97 = 2 × 48 + 1, so it is not a multiple of 2.
This is a contradiction, so no such integers exist.

Answer: Proved

10 Quadratic in sin or cos (All boards)

Solve $2\cos^2 x + \cos x = 0$ for $0° \leq x \leq 360°$.

Factorise: (2 cos x + 1)cos x = 0 (as a quadratic in cos x)

$\cos x = -\frac{1}{2}$ or cos x = 0

x = 90°, 120°, 240°, 270°

Answer: x = 90°, 120°, 240°, 270°