Day 330 **A Level Maths: C Tier 10-a-day: Model Solutions**

1 Parametric equations: finding points (Edexcel, OCR, MEI, CIE, AQA, Eduqas)

A curve has parametric equations

$x = t + 2$, $y = 2t^2 - 1$.

Find the coordinates of the point where $t = -2$.

$x = 1 \times (-2) + 2 = 0$

$y = 2 \times (-2)^2 - 1 = 7$

Answer: (0, 7)

2 3D vectors: distance, magnitude and midpoint (All boards)

A has coordinates $(-1, 4, -3)$ and B has coordinates $(2, -2, -5)$.

Find (a) the vector AB, (b) the distance AB, (c) the midpoint of AB.

(a) $\text{AB} = \text{OB} - \text{OA} = \begin{pmatrix} 3 \\ -6 \\ -2 \end{pmatrix}$

(b) $|\text{AB}| = \sqrt{3^2 + 6^2 + 2^2} = 7$

(c) Midpoint $= (\frac{1}{2}, 1, -4)$

Answer: (a) $\begin{pmatrix} \mathbf{3} \\ \mathbf{-6} \\ \mathbf{-2} \end{pmatrix}$ **(b) 7 (c)** $(\frac{\mathbf{1}}{\mathbf{2}}, \mathbf{1}, \mathbf{-4})$

3 Integrating acceleration to find velocity (AQA, Edexcel, OCR, MEI, Eduqas, CIE)

A particle moves in a straight line. Its acceleration at time t seconds is $(t - 5)$ m s^{-2}.

When $t = 0$ its velocity is 6 m s^{-1}.

Find its velocity when $t = 4$.

$v = \int a \, dt = 0.5t^2 - 5t + c$

$t = 0$, $v = 6$ gives $c = 6$

When $t = 4$: $v = 8 + -20 + 6$

Answer: -6 m s^{-1}

4 Find the angle of a sector (All boards)

A sector has radius 5 cm and area 22.5 cm^2.

Find the angle of the sector in radians and the arc length.

$\frac{1}{2} \times 5^2 \times \theta = 22.5$, so $\theta = 1.8$

Arc length $= r\theta = 5 \times 1.8 = 9.00$ cm

Answer: θ = 1.8 radians; arc length 9.00 cm

5 Histograms (AQA, Edexcel, OCR, MEI, Eduqas, CIE)

The histogram shows the times, t minutes, taken by some people to complete a puzzle.

Find the number of people who took between 20 and 40 minutes.

Frequency = frequency density × class width

$20 \leq t < 30$: $2 \times 10 = 20$

$30 \leq t < 40$: $8 \times 10 = 80$

Total = 20 + 80

Answer: 100

6 Proof by contradiction (AQA, Edexcel, OCR, MEI, Eduqas)

Prove by contradiction that there are no integers x and y such that $10x + 20y = 47$.

Assume that integers x and y exist with $10x + 20y = 47$.

Then $5(2x + 4y) = 47$, so 47 would be a multiple of 5.

But $47 = 5 \times 9 + 2$, so it is not a multiple of 5.

This is a contradiction, so no such integers exist.

Answer: Proved

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7 Hypothesis test for correlation (AQA, Edexcel, OCR, MEI, Eduqas)

15 pairs of data on height and arm span have $r = -0.36$.
Test, at the 5% significance level, for evidence of negative correlation. The critical value is -0.4409.

$H_0: \rho = 0$, $H_1: \rho < 0$
$r = -0.36$ is not beyond the critical value -0.4409
Do not reject H_0
Answer: Do not reject H_0: there is not enough evidence of negative correlation

8 Catching up: constant speed against acceleration (AQA, Edexcel, OCR, MEI, Eduqas, CIE)

Car A passes a point O at constant speed 5 m s^{-1}.
At that instant car B starts from rest at O with constant acceleration 1 m s^{-2} in the same direction. Find the time when B catches up A.

Distances equal: $5t = \frac{1}{2} \times 1 \times t^2$
$t(5 - 0.5t) = 0$, so $t = 0$ or $t = 5 \div 0.5$
Answer: 10 s

9 Angle between vectors (scalar product) (CIE)

$\mathbf{a} = 4\mathbf{i} + \mathbf{j} - \mathbf{k}$ and $\mathbf{b} = 2\mathbf{i} + 2\mathbf{j} - 3\mathbf{k}$.
Find the angle between a and b, to 1 decimal place.

$\mathbf{a}.\mathbf{b} = (4)(2) + (1)(2) + (-1)(-3) = 13$
$|\mathbf{a}| = \sqrt{18}$, $|\mathbf{b}| = \sqrt{17}$
$\cos\theta = \frac{13}{\sqrt{18} \times \sqrt{17}}$, so $\theta = 42.0°$
Answer: 42.0°

10 Equation of a circle (AQA, Edexcel, OCR, MEI, Eduqas, CIE)

A is $(-1, 1)$ and B is $(7, 1)$. AB is a diameter of a circle.
Find the equation of the circle.

Centre = midpoint of AB = $(3, 1)$
$r^2 = (7 - 3)^2 + (1 - 1)^2 = 16$
Equation: $(x - a)^2 + (y - b)^2 = r^2$
Answer: $(x - 3)^2 + (y - 1)^2 = 16$