Day 33 **A Level Maths: C Tier 10-a-day: Model Solutions**

1 Linear inequalities (AQA, Edexcel, OCR, MEI, Eduqas, CIE)

Solve $11 \leq 3x - 1 \leq 23$.

Subtract -1 from each part: $12 \leq 3x \leq 24$

Divide each part by 3.

Answer: $4 \leq x \leq 8$

2 Maximum friction on a horizontal surface (All boards)

A box of mass 5 kg rests on rough horizontal ground. The coefficient of friction is 0.4.

Taking g = 9.8 m s^{-2}, find the maximum friction force that can act on the box.

$R = mg = 5 \times 9.8 = 49$

$F_{max} = \mu R = 0.4 \times 49$

Answer: 19.6 N

3 Double angle: find sin 2A and cos 2A (All boards)

A is acute and $\sin A = \frac{15}{17}$.

Find the exact values of sin 2A and cos 2A.

$\cos A = \sqrt{1 - \sin^2 A} = \frac{8}{17}$

$\sin 2A = 2 \sin A \cos A = 2 \times \frac{15}{17} \times \frac{8}{17} = \frac{240}{289}$

$\cos 2A = \cos^2 A - \sin^2 A = \frac{64}{289} - \frac{225}{289} = -\frac{161}{289}$

Answer: $\sin 2A = \frac{240}{289}$, $\cos 2A = -\frac{161}{289}$

4 Equation of a circle (AQA, Edexcel, OCR, MEI, Eduqas, CIE)

A is $(-2, -6)$ and B is $(8, 4)$. AB is a diameter of a circle.

Find the equation of the circle.

Centre = midpoint of AB = $(3, -1)$

$r^2 = (8 - 3)^2 + (4 - (-1))^2 = 50$

Equation: $(x - a)^2 + (y - b)^2 = r^2$

Answer: $(x - 3)^2 + (y + 1)^2 = 50$

5 Vertical motion: greatest height from speed (AQA, Edexcel, OCR, MEI, Eduqas, CIE)

A ball is projected vertically upwards with speed 14 m s^{-1} from ground level. Taking g = 9.8 m s^{-2}, find the maximum height it reaches above the ground.

At the top $v = 0$: $v^2 = u^2 + 2as$

$0 = 14^2 - 2 \times 9.8 \times s$

$s = \frac{196}{19.6}$

Answer: 10 m

6 Grouped data: estimated median (AQA, Edexcel, OCR, MEI, Eduqas, CIE)

The table shows grouped data.

Class: $0 \leq x < 10$; $10 \leq x < 20$; $20 \leq x < 30$; $30 \leq x < 40$

Frequency: 11, 5, 3, 8

Estimate the median, to 3 s.f.

n = 27, so the median is the 13.5 th value, in class $10 \leq x < 20$

Cumulative frequency before this class = 11

$\text{Median} \approx 10 + \frac{(13.5 - 11)}{5} \times 10$

Answer: 15.0

7 Implicit differentiation (AQA, Edexcel, OCR, MEI, Eduqas, CIE)

A curve has equation $x^2 + y^2 = 25$.
Find dy/dx in terms of x and y, and hence the gradient at (3, 4).

Differentiate both sides with respect to x (using dy/dx for y terms)

$dy/dx = -\frac{x}{y}$

At (3, 4): $dy/dx = -\frac{3}{4}$

Answer: $dy/dx = -\frac{x}{y}$; gradient $-\frac{3}{4}$

8 Conditional probability (Bayes) (AQA, Edexcel, OCR, MEI, Eduqas, CIE)

The probability that a machine is set up wrongly is 0.6. If so, the probability that an item is faulty is 0.7; otherwise it is 0.5. Given that an item is faulty, find the probability that a machine is set up wrongly.

P(both) = 0.6 × 0.7 = 0.42; P(not first and second) = 0.4 × 0.5 = 0.2
P(second) = 0.42 + 0.2 = 0.62
P = 0.42 ÷ 0.62

Answer: $\frac{21}{31}$ = 0.677

9 Factorised trig equation (All boards)

Solve $\sin x(2 \sin x - 1) = 0$ for $0° \leq x \leq 360°$.

Either $\sin x = 0$ or $2 \sin x - 1 = 0$
$\sin x = 0$: x = 0°, 180°, 360°

$\sin x = \frac{1}{2}$: x = 30°, 150°

Answer: x = 0°, 30°, 150°, 180°, 360°

10 Recurrence relations (AQA, Edexcel, OCR, MEI, Eduqas, CIE)

A sequence is defined by $u_{n+1} = -u_n - 4$, $u_1 = -1$.
Find u_2, u_3 and u_4.

$u_2 = (-1) \times (-1) - 4 = -3$
$u_3 = (-1) \times (-3) - 4 = -1$
$u_4 = (-1) \times (-1) - 4 = -3$

Answer: $u_2 = -3$, $u_3 = -1$, $u_4 = -3$