Day 329 **A Level Maths: C Tier 10-a-day: Model Solutions**

1 Centre and radius of a circle (AQA, Edexcel, OCR, MEI, Eduqas, CIE)

A circle has equation $x^2 + y^2 + 12x - 8y + 7 = 0$.
Find the centre and the radius of the circle.

Complete the square in x and in y:

$(x + 6)^2 + (y - 4)^2 = 45$

Centre $(-6, 4)$, radius $= 3\sqrt{5}$

Answer: Centre $(-6, 4)$, radius $3\sqrt{5}$

2 Counting: arrangements and selections (CIE)

In how many ways can a team of 2 be chosen from 9 members?

$^9C_2 = \frac{9\times8}{2\times1}$

Answer: 36

3 Trapezium rule from a table (AQA, Edexcel, OCR, MEI, Eduqas)

The table shows values of y = f(x).
x: 0 0.5 1 1.5 2 2.5
y: 7.6 8.5 3.7 3.3 8.3 2.8
Use the trapezium rule to estimate the area under the curve between x = 0 and x = 2.5.

h = 0.5 with 5 strips

Area $\approx \frac{0.5}{2}[7.6 + 2.8 + 2(8.5 + 3.7 + 3.3 + 8.3)]$

$= \frac{0.5}{2} \times 58$

$= 14.5$

Answer: 14.5

4 Moments: rod on a pivot with a particle (AQA, Edexcel, OCR, MEI)

A uniform rod AB of mass 4 kg and length 4 m is supported by a pivot 0.5 m from A. A particle of mass m kg is attached at A and the rod rests horizontally in equilibrium. Find m.

Moments about the pivot:

$m\,g \times 0.5 = 4\,g \times (2 - 0.5)$

$m = \frac{4 \times 1.5}{0.5}$

Answer: 12 kg

5 Continuous random variable: probability and mean (CIE)

X has probability density function $f(x) = \frac{3}{64}x^2$ for $0 \le x \le 4$, and 0 otherwise.

Find $P(X < 2)$.

$P(X < 2) = \int$ from 0 to 2 of $\frac{3}{64}x^2\,dx$

$= [\frac{3}{192}x^3]$ from 0 to 2

Answer: $\frac{1}{8}$

6 Angle between vectors (scalar product) (CIE)

$\mathbf{a} = 3\mathbf{i} + 3\mathbf{j} - 3\mathbf{k}$ and $\mathbf{b} = 2\mathbf{i} + 2\mathbf{k}$.
Find the angle between a and b, to 1 decimal place.

$\mathbf{a}.\mathbf{b} = (3)(2) + (3)(0) + (-3)(2) = 0$

$|\mathbf{a}| = \sqrt{27}$, $|\mathbf{b}| = \sqrt{8}$

$\cos\theta = \frac{0}{\sqrt{27} \times \sqrt{8}}$, so $\theta = 90.0°$

Answer: 90.0°

Day 329 **A Level Maths: C Tier 10-a-day: Model Solutions**

7 Geometric series: sum to infinity (AQA, Edexcel, OCR, MEI, Eduqas, CIE)

A geometric series has first term 14 and common ratio $\frac{3}{4}$.

Find the sum to infinity.

$|r| < 1$, so the series converges.

$$S_\infty = \frac{a}{1-r} = \frac{14}{1-\frac{3}{4}}$$

$= 56$

Answer: 56

8 Braking: deceleration and time (AQA, Edexcel, OCR, MEI, Eduqas, CIE)

A lorry travelling at 24 m s^{-1} brakes uniformly and comes to rest after travelling 120 m.

(a) Find the deceleration.

(b) Find the time taken to stop.

(a) $v^2 = u^2 + 2as$: $0 = 24^2 + 2 \times a \times 120$

so $a = -2.4$, a deceleration of 2.4

(b) $s = \frac{(u+v)}{2}t$: $120 = 12\,t$

Answer: (a) 2.4 m s^{-2} (b) 10 s

9 Geometric series: sum of n terms (AQA, Edexcel, OCR, MEI, Eduqas, CIE)

Find the sum of the first 9 terms of the geometric series $2 + 4 + 8 + \ldots$

$a = 2$, $r = 2$, $n = 9$

$$S_n = \frac{a(r^n - 1)}{r - 1} = \frac{2(2^9 - 1)}{2 - 1}$$

$= 1022$

Answer: 1022

10 Newton-Raphson: one or two steps (AQA, Edexcel, OCR, MEI, Eduqas)

$f(x) = x^3 - 4x - 8$.

Use the Newton-Raphson method with $x_0 = 3$ to find x_1,

giving your answer to 3 decimal places.

$f'(x) = 3x^2 - 4$

$$x_{n+1} = x_n - \frac{f(x_n)}{f'(x_n)}$$

$$x_1 = 3 - \frac{7.000}{23.00} = 2.6957$$

Answer: $x_1 = 2.696$