Day 326 **A Level Maths: C Tier 10-a-day: Model Solutions**

1 Natural logs and e^x (All boards)

Solve (a) $e^x = 20$ (b) $\ln x = 2.5$
Give answers to 3 significant figures.

(a) $x = \ln 20 = 3.00$

(b) $x = e^{2.5} = 12.2$

Answer: (a) x = 3.00 (b) x = 12.2

2 Hypotheses for a binomial test (AQA, Edexcel, OCR, MEI, Eduqas, CIE)

A player says that she wins a game with probability 0.75.
A researcher believes that the probability is lower. Let p be the probability that she wins a game.
Write down suitable hypotheses for a test on p.

H_0 states the claimed value; H_1 matches the researcher's belief.

One-tailed test

Answer: H_0: p = 0.75, H_1: p < 0.75

3 Change of sign to locate a root (All boards)

$f(x) = x^2 - \ln x - 6$.
Show that f(x) = 0 has a root between x = 2 and x = 3.

f(2) = −2.69
f(3) = 1.90
f is continuous and there is a change of sign, so a root lies in the interval

Answer: f(2) = −2.69, f(3) = 1.90: change of sign

4 Finding an unknown using the factor theorem (AQA, Edexcel, OCR, MEI, Eduqas, CIE)

$(x - 3)$ is a factor of $x^3 + kx^2 - 4x - 60$.
Find the value of k.

Let $f(x) = x^3 + kx^2 - 4x - 60$. Then f(3) = 0:
$27 + 9k - 72 = 0$
$9k = 45$

Answer: k = 5

5 Stone thrown down from a cliff (AQA, Edexcel, OCR, MEI, Eduqas, CIE)

A stone is thrown vertically downwards at 12 m s^{-1} from the top of a cliff and hits the sea after 2 s. Taking g = 9.8 m s^{-2}, find the height of the cliff.

$s = ut + \frac{1}{2}at^2$

$s = 12 \times 2 + \frac{1}{2} \times 9.8 \times 2^2$

$s = 24 + 19.6$

Answer: 43.6 m

6 Separable differential equations (All boards)

Given that dy/dx = 5xy and y = 3 when x = 0,
find y in terms of x.

$\frac{1}{y}\,dy = 5x\,dx$, so $\ln y = \frac{5}{2}x^2 + c$

x = 0, y = 3: $c = \ln 3$

$y = 3\,e^{\frac{5x^2}{2}}$

Answer: $y = 3\,e^{\frac{5x^2}{2}}$

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7 Solving equations with indices (AQA, Edexcel, OCR, MEI, Eduqas, CIE)

Solve $27^x = 81$.

$27 = 3^3$ and $81 = 3^4$

$3^{3x} = 3^4$, so $3x = 4$

Answer: $x = \frac{4}{3}$

8 Stratified sampling (AQA, Edexcel, OCR, MEI, Eduqas)

A stratified sample of size 16 is taken from 112 employees: 70 Primary, 42 Secondary.
How many Primary should be in the sample?

Number $= 16 \times \frac{70}{112}$

$= 10$

Answer: 10

9 Velocity-time graph: acceleration from two stages (AQA, Edexcel, OCR, MEI, Eduqas, CIE)

The graph shows the velocity of a skater.
(a) Find the acceleration in the first 8 s.
(b) Find the deceleration in the last 3 s.
(c) Find the total distance travelled.

(a) $a = \frac{9 - 5}{8}$

(b) $a = \frac{9 - 0}{3}$, a deceleration of 3

(c) area $= \frac{1}{2}(5 + 9) \times 8 + 9 \times 12 + \frac{1}{2} \times 3 \times 9$

Answer: (a) 0.5 m s^{-2} (b) 3 m s^{-2} (c) 177.5 m

10 Proof by contradiction (AQA, Edexcel, OCR, MEI, Eduqas)

Prove by contradiction that if $3n + 10$ is even, then n is even, where n is an integer.

Assume n is odd, so $n = 2k + 1$ for an integer k.
$3n + 10 = 6k + 3 + 10 = 2(3k + 6) + 1$
This is odd, which contradicts $3n + 10$ being even.
So the assumption is false: n must be even.
Answer: Proved