Day 325 **A Level Maths: C Tier 10-a-day: Model Solutions**

1 Equation of a circle from a diagram (AQA, Edexcel, OCR, MEI, Eduqas, CIE)

The diagram shows a circle with the centre marked.
The circle touches the x-axis. Find the equation of the circle.

The circle touches the x-axis, so the radius equals the y-coordinate of the centre: r = 2.

Centre (0, 2): $(x - a)^2 + (y - b)^2 = r^2$

Answer: $x^2 + (y - 2)^2 = 4$

2 Inverse trig functions (Edexcel, OCR)

Find the exact value, in radians, of

(a) $\arccos(\frac{1}{2})$ (b) $\arccos(1)$

(a) $y = \arccos(\frac{1}{2})$ means $\cos y = \frac{1}{2}$ with $0 \leq y \leq \pi$: $y = \frac{\pi}{3}$

(b) $y = \arccos(1)$ means $\cos y = 1$ with $0 \leq y \leq \pi$: $y = 0$

Answer: (a) $\frac{\pi}{3}$ (b) 0

3 Describing transformations of graphs (AQA, Edexcel, OCR, MEI, Eduqas, CIE)

Describe the single transformation that maps the graph of y = f(x) onto the graph of $y = \frac{1}{6}f(x)$.

$y = \frac{1}{6}f(x)$ is obtained from y = f(x) by a:

stretch parallel to the y-axis, scale factor $\frac{1}{6}$

Answer: stretch parallel to the y-axis, scale factor $\frac{1}{6}$

4 Tree diagrams (AQA, Edexcel, OCR, MEI, Eduqas, CIE)

The tree diagram shows probabilities for events R and L.
Find the probability that exactly one of R and L happens.

Exactly one: (R and L′) or (R′ and L)
= 0.3 × 0.1 + 0.7 × 0.4
Answer: 0.31

5 Power of an engine: driving force (CIE)

A lorry's engine works at a constant rate of 12 kW. Find the driving force when the vehicle is moving at 10 m s^{-1}.

P = Fv
12000 = F × 10
Answer: 1200 N

6 Separable differential equations (All boards)

Given that $dy/dx = 4x^2$ and y = 2 when x = 0,
find y in terms of x.

$y = \int 4x^2\,dx = \frac{4x^3}{3} + c$

x = 0, y = 2: c = 2

Answer: $y = \frac{4x^3}{3} + 2$

Day 325 **A Level Maths: C Tier 10-a-day: Model Solutions**

7 Tangent condition for a line and a curve (AQA, Edexcel, OCR, MEI, Eduqas, CIE)

The line $y = -x + k$ is a tangent to the curve $y = x^2 + 3x - 4$.
Find the value of k.

$x^2 + 3x - 4 = -x + k$
$x^2 + 4x + (-4 - k) = 0$
A tangent touches once: discriminant = 0
$(4)^2 - 4(-4 - k) = 0$
Answer: $k = -8$

8 Change of sign to locate a root (All boards)

$f(x) = x^3 + 4x - 6$.
Show that $f(x) = 0$ has a root between $x = 1$ and $x = 1.5$.

$f(1) = -1.00$
$f(1.5) = 3.38$
f is continuous and there is a change of sign, so a root lies in the interval
Answer: $f(1) = -1.00$, $f(1.5) = 3.38$: change of sign

9 Continuous random variable: finding k (CIE)

The continuous random variable X has probability density function $f(x) = kx$ for $0 \leq x \leq 3$, and 0 otherwise.
Find the value of k.

$\int$ from 0 to 3 of $k\,x\,dx = 1$
$k \times \frac{3^2}{2} = 1$
Answer: $k = \frac{2}{9}$

10 Two-stage motion: acceleration then constant speed (AQA, Edexcel, OCR, MEI, Eduqas, CIE)

A train starts from rest and accelerates uniformly at 2 m s^{-2} for 10 s, then travels at constant speed for a further 10 s. Find the total distance travelled.

Speed reached: $v = 2 \times 10 = 20$ m s^{-1}
Stage 1: $s = \frac{1}{2} \times 20 \times 10 = 100$
Stage 2: $s = 20 \times 10 = 200$
Answer: 300 m