Day 324 **A Level Maths: C Tier 10-a-day: Model Solutions**

1 Vector equation of a line (3D) (CIE)

A line has equation $\mathbf{r} = \begin{pmatrix} 1 \\ -2 \\ 4 \end{pmatrix} + t\begin{pmatrix} 3 \\ -1 \\ 2 \end{pmatrix}$.

Write down a vector equation of the line through the origin, parallel to this line.

Parallel lines have the same direction vector

The line passes through O, so the position vector is the zero vector

Answer: $\mathbf{r} = s\begin{pmatrix} 3 \\ -1 \\ 2 \end{pmatrix}$

2 Standard deviation from summary data (AQA, Edexcel, OCR, MEI, Eduqas, CIE)

For 25 values, $\Sigma x = 300$ and $\Sigma x^2 = 3800$.

Find the mean and the standard deviation.

Mean $= 300 \div 25 = 12$

Variance $= \frac{3800}{25} - 12^2 = 152 - 144 = 8$

Standard deviation $= \sqrt{8}$

Answer: mean = 12, standard deviation = 2.83

3 Friction: acceleration along rough ground (All boards)

A block of mass 10 kg is pulled along rough horizontal ground by a horizontal force of 54.2 N.

The coefficient of friction is 0.4. Take $g = 9.8$ m s^{-2}. Find the acceleration.

$R = 10 \times 9.8 = 98$

$F = \mu R = 0.4 \times 98 = 39.2$

$P - F = ma$: $54.2 - 39.2 = 10a$

Answer: 1.5 m s^{-2}

4 Expanding brackets with surds (AQA, Edexcel, OCR, MEI, Eduqas, CIE)

Expand and simplify $(6 + \sqrt{5})(6 - \sqrt{5})$.

$= 6^2 - (\sqrt{5})^2$

$= 36 - 5$

Answer: 31

5 Continuous random variable: finding k (CIE)

The continuous random variable X has probability density function $f(x) = k(x + 2)$ for $0 \le x \le 3$, and 0 otherwise.

Find the value of k.

$\int$ from 0 to 3 of $k(x + 2)\,dx = 1$

$k[\frac{x^2}{2} + 2x]$ from 0 to 3 $= k(\frac{21}{2}) = 1$

Answer: $\mathbf{k = \frac{2}{21}}$

6 Addition formulae: exact values (All boards)

By writing 105° as 45° + 60°, find the exact value of cos 105°.

$\cos(45° + 60°) = \cos 45° \cos 60° - \sin 45° \sin 60°$

$= \frac{\sqrt{2}}{2} \times \frac{1}{2} - \frac{\sqrt{2}}{2} \times \frac{\sqrt{3}}{2}$

Simplify: $-\frac{\sqrt{6} - \sqrt{2}}{4}$

Answer: $\mathbf{-\frac{\sqrt{6} - \sqrt{2}}{4}}$

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7 Perpendicular bisector (AQA, Edexcel, OCR, MEI, Eduqas, CIE)

A is (0, −1) and B is (6, −5).

Find the equation of the perpendicular bisector of AB, in the form ax + by = c with integer a, b and c.

Midpoint of AB = (3, −3)

Gradient of AB = $\frac{-4}{6} = -\frac{2}{3}$, so perpendicular gradient = $\frac{3}{2}$

$y - (-3) = \frac{3}{2}(x - 3)$

Multiply out and rearrange.

Answer: 3x − 2y = 15

8 Estimate with a small angle (AQA, Edexcel, OCR, MEI, Eduqas)

Use $\sin\theta \approx \theta$ and $\cos\theta \approx 1 - \frac{\theta^2}{2}$ to estimate $\frac{\sin 0.08}{\cos 0.08}$.

$$\frac{\sin\theta}{\cos\theta} \approx \frac{\theta}{1 - \frac{\theta^2}{2}}$$

$$= \frac{0.08}{1 - 0.00320} = \frac{0.08}{0.9968} = 0.0803$$

Answer: 0.0803

9 Combining two inequalities (AQA, Edexcel, OCR, MEI, Eduqas, CIE)

Find the set of values of x satisfying both

$2x - 3 > 3$ and $x^2 - 8x + 12 < 0$.

$2x - 3 > 3$ gives $x > 3$.

$x^2 - 8x + 12 < 0$: $(x - 2)(x - 6) < 0$, so $2 < x < 6$.

Both are true where the two ranges overlap.

Answer: $3 < x < 6$

10 Work done and kinetic energy (CIE)

A particle of mass 4 kg changes its speed from 1 m s^{-1} to 6 m s^{-1}. Find the increase in its kinetic energy.

$KE = \frac{1}{2}mv^2$

Increase = $\frac{1}{2} \times 4 \times 6^2 - \frac{1}{2} \times 4 \times 1^2$

Answer: 70 J