Day 322 **A Level Maths: C Tier 10-a-day: Model Solutions**

1 Simplify using identities (All boards)

Simplify $1 - \cos^2 x$.

$\sin^2 x + \cos^2 x = 1$, so $1 - \cos^2 x = \sin^2 x$

Answer: $\sin^2 x$

2 Quadratic inequalities (AQA, Edexcel, OCR, MEI, Eduqas, CIE)

Solve $x + 4 \leq 12$.

Solve $x^2 + 4x - 12 = 0$: $(x + 6)(x - 2) = 0$
Critical values $x = -6$ and $x = 2$.
The graph is a ∪-shaped parabola, so take the region between the roots.
Answer: $-6 \leq x \leq 2$

3 Normal distribution: inverse (AQA, Edexcel, OCR, MEI, Eduqas, CIE)

$X \sim N(50, 10^2)$. Given that $P(X > a) = 0.2$, find a to 3 s.f.

$P(Z > z) = 0.2$, so $z = 0.8416$ (from the inverse normal function)
$a = 50 + 0.8416 \times 10$
Answer: 58.4

4 Solving quadratics: surd answers (AQA, Edexcel, OCR, MEI, Eduqas, CIE)

Solve $x^2 + 4x + 1 = 0$, giving your answers in the form $p \pm \sqrt{q}$.

Complete the square: $(x + 2)^2 - 4 + 1 = 0$
$(x + 2)^2 = 3$
$x + 2 = \pm\sqrt{3}$
Answer: $x = -2 \pm \sqrt{3}$

5 Newton-Raphson: one or two steps (AQA, Edexcel, OCR, MEI, Eduqas)

$f(x) = x^3 - 4x - 6$.
Use the Newton-Raphson method with $x_0 = 2$ to find x_1,
giving your answer to 3 decimal places.

$f'(x) = 3x^2 - 4$

$$x_{n+1} = x_n - \frac{f(x_n)}{f'(x_n)}$$

$$x_1 = 2 - \frac{-6.000}{8.000} = 2.7500$$

Answer: $x_1 = 2.750$

6 Catching up: constant speed against acceleration (AQA, Edexcel, OCR, MEI, Eduqas, CIE)

Car P passes a point O at constant speed 6 m s^{-1}.
At that instant car Q starts from rest at O with constant acceleration 1.5 m s^{-2} in the same direction. Find the time when Q catches up P.

Distances equal: $6t = \frac{1}{2} \times 1.5 \times t^2$
$t(6 - 0.75t) = 0$, so $t = 0$ or $t = 6 \div 0.75$
Answer: 8 s

7 Two-tailed binomial test (AQA, Edexcel, OCR, MEI, Eduqas, CIE)

$X \sim B(12, p)$. H_0: $p = 0.3$, H_1: $p \neq 0.3$. In a sample, $x = 6$.
Test at the 10% significance level.

Two-tailed test, so compare with $0.1 \div 2 = 0.05$
$P(X \geq 6) = 0.118$
$0.118 > 0.05$, so do not reject H_0
Answer: Do not reject H_0 ($P(X \geq 6) = 0.118$)

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8 Solve a logarithmic equation (two logs) (All boards)

Solve $\log x + \log (x - 1) = \log 30$, $x > 1$.

$\log [x(x - 1)] = \log 30$, so $x^2 - 1x = 30$
$x^2 - 1x - 30 = 0$, $(x - 6)(x + 5) = 0$
$x = 6$ or $x = -5$; $x > 1$, so $x = 6$
Answer: $x = 6$

9 Proof by contradiction (AQA, Edexcel, OCR, MEI, Eduqas)

Prove by contradiction that there are no integers x and y such that $12x + 9y = 74$.

Assume that integers x and y exist with $12x + 9y = 74$.
Then $3(4x + 3y) = 74$, so 74 would be a multiple of 3.
But $74 = 3 \times 24 + 2$, so it is not a multiple of 3.
This is a contradiction, so no such integers exist.
Answer: Proved

10 Particle held by a string and a horizontal force (All boards)

A lamp of weight 60 N hangs from a light string, pulled aside by a horizontal force P N so that the string makes an angle θ with the vertical, where $\tan \theta = \frac{5}{12}$. Find P and the tension T N.

Vertically: $T \cos \theta = 60$
Horizontally: $T \sin \theta = P$
Dividing: $P = 60 \tan \theta$; $T = \sqrt{60^2 + P^2}$
Answer: $P = 25$, $T = 65$