Day 32 **A Level Maths: C Tier 10-a-day: Model Solutions**

1 Conditional probability formula (AQA, Edexcel, OCR, MEI, Eduqas, CIE)

P(A ∩ B) = 0.12 and P(B) = 0.3.
Find P(A | B).

P(A | B) = P(A ∩ B) ÷ P(B)
= 0.12 ÷ 0.3
Answer: 0.4

2 Braking: deceleration and time (AQA, Edexcel, OCR, MEI, Eduqas, CIE)

A cyclist travelling at 30 m s^{-1} brakes uniformly and comes to rest after travelling 90 m.
(a) Find the deceleration.
(b) Find the time taken to stop.

(a) $v^2 = u^2 + 2as$: $0 = 30^2 + 2 \times a \times 90$
so $a = -5$, a deceleration of 5

(b) $s = \frac{(u + v)}{2} t$: $90 = 15 t$

Answer: (a) 5 m s^{-2} (b) 6 s

3 Geometric sequences (AQA, Edexcel, OCR, MEI, Eduqas, CIE)

A geometric sequence begins 2, 6, 18, ...
Find the common ratio and the 5th term.

$r = \frac{6}{2} = 3$

$u_n = ar^{n-1}$, so $u_5 = 2 \times (3)^4$
$= 162$
Answer: r = 3, u_5 = 162

4 Differentiate tan, sec, cosec, cot (AQA, Edexcel, OCR, MEI, Eduqas)

Differentiate y = cot 4x with respect to x.

Use the standard result: d/dx(cot kx) = $-k \operatorname{cosec}^2 kx$
Answer: dy/dx = $-4 \operatorname{cosec}^2 4x$

5 Completing the square (AQA, Edexcel, OCR, MEI, Eduqas, CIE)

Write $3x^2 + 18x + 32$ in the form $a(x + p)^2 + q$.

Take out a factor 3: $3(x^2 + 6x + \frac{32}{3})$

$x + 6 = (x + 3)^2 - 9$
$3[(x + 3)^2 - 9] + 32 = 3(x + 3)^2 + 5$
Answer: $3(x + 3)^2 + 5$

6 Binomial: finding parameters (AQA, Edexcel, OCR, MEI, Eduqas, CIE)

X ~ B(100, p) and E(X) = 30.
Find p and Var(X).

np = 30, so p = 30 ÷ 100 = 0.3
Var(X) = npq = 30 × 0.7
Answer: p = 0.3, Var(X) = 21

7 Sigma notation (AQA, Edexcel, OCR, MEI, Eduqas, CIE)

Evaluate $\sum_{r=1}^{10} (-2r + 3)$.

This is an arithmetic series with first term 1 and common difference −2.

$S_{10} = \frac{10}{2}[2 \times 1 + 9 \times (-2)]$
$= -80$
Answer: −80

8 Partial fractions: find the constants (All boards)

Express $\frac{x+4}{(x+2)(x+1)}$ in the form $\frac{A}{x+2}+\frac{B}{x+1}$.

Find the values of A and B.

$x + 4 \equiv A(x + 1) + B(x + 2)$

$x = -2$: $2 = A(-1)$, so $A = -2$

$x = -1$: $3 = B(1)$, so $B = 3$

Answer: $A = -2$, $B = 3$

9 Maximum speed from calculus (AQA, Edexcel, OCR, MEI, Eduqas, CIE)

A particle moves in a straight line. Its velocity at time t seconds ($0 \le t \le 6$) is $v = (-4t^2 + 24t)$ m s^{-1}. Find the maximum velocity and the time at which it occurs.

$a = \frac{dv}{dt} = 24 - 8t$

Maximum when $a = 0$: $t = \frac{24}{8} = 3$

$v = 24 \times 3 - 4 \times 3^2$

Answer: 36 m s^{-1} at $t = 3$

10 Identity then solve (sin² to cos) (All boards)

Solve $-\cos^2 x - \sin x + 1 = 0$
for $0° \le x \le 360°$.

Use $\cos^2 x = 1 - \sin^2 x$: $\sin^2 x - \sin x = 0$

Factorise and solve: $\sin x = 1$ or $\sin x = 0$

$x = 0°, 90°, 180°, 360°$

Answer: $x = 0°, 90°, 180°, 360°$