Day 319 **A Level Maths: C Tier 10-a-day: Model Solutions**

1 Trapezium rule from a table (AQA, Edexcel, OCR, MEI, Eduqas)

The table shows values of y = f(x).
x: 0 0.25 0.5 0.75 1 1.25
y: 7.5 8.1 7.6 1.7 1.9 1.6
Use the trapezium rule to estimate the area under the curve between x = 0 and x = 1.25.

h = 0.25 with 5 strips

Area $\approx \frac{0.25}{2}[7.5 + 1.6 + 2(8.1 + 7.6 + 1.7 + 1.9)]$

$= \frac{0.25}{2} \times 47.7$

$= 5.9625$

Answer: 5.9625

2 Differentiating velocity to find acceleration (AQA, Edexcel, OCR, MEI, Eduqas, CIE)

A particle moves in a straight line. Its velocity at time t seconds is v m s^{-1}, where $v = 4t^2 - 16t + 4$.
(a) Find its acceleration when t = 2.
(b) Find the time when its acceleration is zero.

$a = \frac{dv}{dt} = 8t - 16$

(a) t = 2: $a = 8 \times 2 - 16$

(b) $8t - 16 = 0$, so $t = \frac{16}{8}$

Answer: (a) 0 m s^{-2} (b) 2 s

3 Domain and range (AQA, Edexcel, OCR, MEI, Eduqas, CIE)

$f(x) = (x + 1)^2 - 4$, $x \geq 0$.
Find the range of f.

For $x \geq 0$ the least value is at x = 0 (the vertex x = −1 is not allowed).
f(0) = 1 − 4 = −3, and f increases for x > 0.
Answer: f(x) ≥ −3

4 Standard deviation from summary data (AQA, Edexcel, OCR, MEI, Eduqas, CIE)

For 10 values, $\Sigma x = 320$ and $\Sigma x^2 = 10520$.
Find the mean and the standard deviation.

Mean = 320 ÷ 10 = 32

Variance $= \frac{10520}{10} - 32^2 = 1052 - 1024 = 28$

Standard deviation $= \sqrt{28}$

Answer: mean = 32, standard deviation = 5.29

5 Velocity zero (quadratic in t) (AQA, Edexcel, OCR, MEI, Eduqas, CIE)

A particle moves in a straight line. Its velocity at time t seconds is v = $(t^2 - 11t + 24)$ m s^{-1}.
(a) Find the times when the particle is at rest.
(b) Find its acceleration at the first of these times.

(a) $t^2 - 11t + 24 = 0$
$(t - 3)(t - 8) = 0$

(b) $a = \frac{dv}{dt} = 2t - 11$; at t = 3, a = 6 − 11

Answer: (a) t = 3 and t = 8 (b) −5 m s^{-2}

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6 Modulus graphs (AQA, Edexcel, OCR, MEI, Eduqas, CIE)

Sketch $y = |x^2 - 4|$. Write down the coordinates where the graph meets the axes and the coordinates of the local maximum point.

$y = 0$: $x^2 = 4$, so $x = \pm 2$

$x = 0$: $y = |-4| = 4$. The part of $x^2 - 4$ below the x-axis is reflected in it.

The local maximum is at $x = 0$.

Answer: (2, 0), (−2, 0), (0, 4) ; maximum (0, 4)

7 Integrate using partial fractions (All boards)

$\frac{2x + 4}{(x + 4)(x + 3)} \equiv \frac{A}{x + 4} + \frac{B}{x + 3}$.

Find A and B, and hence find $\int \frac{2x + 4}{(x + 4)(x + 3)}\, dx$.

$2x + 4 \equiv A(x + 3) + B(x + 4)$

$x = -4$ gives $A = 4$; $x = -3$ gives $B = -2$

$\int \left(\frac{4}{x + 4} + \frac{-2}{x + 3}\right) dx = 4 \ln|x + 4| + -2 \ln|x + 3| + c$

Answer: $A = 4$, $B = -2$; $4 \ln|x + 4| - 2 \ln|x + 3| + c$

8 Conditional probability (Bayes) (AQA, Edexcel, OCR, MEI, Eduqas, CIE)

The probability that it rains is 0.5. If so, the probability that Sam is late is 0.3; otherwise it is 0.2. Given that Sam is late, find the probability that it rains.

P(both) = 0.5 × 0.3 = 0.15; P(not first and second) = 0.5 × 0.2 = 0.1

P(second) = 0.15 + 0.1 = 0.25

P = 0.15 ÷ 0.25

Answer: $\frac{3}{5} = 0.6$

9 Find a and b in y = ab^x (All boards)

The curve $y = ab^x$ passes through (0, 6) and $(2, \frac{2}{3})$.

Find the values of a and b.

$x = 0$: $y = a \times b^0 = a$, so $a = 6$

$x = 2$: $6 \times b^2 = \frac{2}{3}$, so $b^2 = \frac{1}{9}$

$b = \frac{1}{3}$

Answer: $a = 6$, $b = \frac{1}{3}$

10 Partial fractions (Edexcel, OCR, MEI, CIE, AQA, Eduqas)

Express $\frac{2x - 5}{(x + 3)^2}$ in the form $\frac{A}{(x + 3)} + \frac{B}{(x + 3)^2}$.

$2x - 5 = A(x + 3) + B$

Coefficient of x: $A = 2$

Constant: $-5 = -6 + B$, so $B = -11$

Answer: $A = 2$, $B = -11$