Day 317 **A Level Maths: C Tier 10-a-day: Model Solutions**

1 Starting from rest: find the time (AQA, Edexcel, OCR, MEI, Eduqas, CIE)

A skier starts from rest and moves in a straight line with constant acceleration 0.5 m s^{-2}.
It travels 16 m. Find the time taken.

$s = ut + \frac{1}{2}at^2$ with $u = 0$

$16 = \frac{1}{2} \times 0.5 \times t^2$

$t^2 = 64$

Answer: 8 s

2 2D vectors: magnitude and direction (All boards)

The vector $\mathbf{p} = -12\mathbf{i} - 5\mathbf{j}$.
Find (a) |p|, (b) the angle p makes with the positive x-axis, measured anticlockwise (1 d.p.).

(a) $|\mathbf{p}| = \sqrt{12^2 + 5^2} = 13$

(b) Acute angle $= \tan^{-1}(\frac{5}{12}) = 22.6°$

p is in the third quadrant, so the angle is 202.6°

Answer: (a) 13 (b) 202.6°

3 Real roots and the discriminant (AQA, Edexcel, OCR, MEI, Eduqas, CIE)

Find the set of values of k for which
$2x^2 + 4x + k = 0$ has no real roots.

$b^2 - 4ac < 0$
$16 - 8k < 0$

Answer: $k > 2$

4 3D vectors: distance, magnitude and midpoint (All boards)

$\mathrm{OA} = \begin{pmatrix} 3 \\ 3 \\ -1 \end{pmatrix}$ and $\mathrm{OB} = \begin{pmatrix} 0 \\ -9 \\ -5 \end{pmatrix}$.
Find AB and |AB|, and a unit vector in the direction of AB.

$\mathrm{AB} = \mathrm{OB} - \mathrm{OA} = \begin{pmatrix} -3 \\ -12 \\ -4 \end{pmatrix}$

$|\mathrm{AB}| = \sqrt{3^2 + 12^2 + 4^2} = 13$

Unit vector $= \frac{1}{13}$ AB

Answer: $\mathbf{AB} = \begin{pmatrix} -3 \\ -12 \\ -4 \end{pmatrix}$, $|\mathbf{AB}| = 13$, unit vector $\frac{1}{13}\begin{pmatrix} -3 \\ -12 \\ -4 \end{pmatrix}$

5 Hypotheses for a binomial test (AQA, Edexcel, OCR, MEI, Eduqas, CIE)

A company says that the probability a customer buys a ticket is 0.75.
A researcher believes that the probability is higher. Let p be the probability that customer buys a ticket.
Write down suitable hypotheses for a test on p.

H_0 states the claimed value; H_1 matches the researcher's belief.

One-tailed test

Answer: H_0: $p = 0.75$, H_1: $p > 0.75$

6 Finding an unknown using the factor theorem (AQA, Edexcel, OCR, MEI, Eduqas, CIE)

When $x^3 + kx^2 - 4x + 16$ is divided by $(x - 2)$, the remainder is 8. Find the value of k.

$f(2) = 8$
$8 + 4k + 8 = 8$
$4k = -8$

Answer: $k = -2$

Day 317 **A Level Maths: C Tier 10-a-day: Model Solutions**

7 Catching up: constant speed against acceleration (AQA, Edexcel, OCR, MEI, Eduqas, CIE)

Car A passes a point O at constant speed 3 m s^{-1}.
At that instant car B starts from rest at O with constant acceleration 1 m s^{-2} in the same direction. Find the time when B catches up A.

Distances equal: $3t = \frac{1}{2} \times 1 \times t^2$

$t(3 - 0.5t) = 0$, so $t = 0$ or $t = 3 \div 0.5$

Answer: 6 s

8 Gradient and parallel or perpendicular lines (AQA, Edexcel, OCR, MEI, Eduqas, CIE)

The line $y = -\frac{1}{2}x + 3$ is perpendicular to the line
$y = kx - 5$. Find the value of k.

Perpendicular lines have gradients with product -1.

$(-\frac{1}{2}) \times k = -1$

Answer: k = 2

9 Critical region (AQA, Edexcel, OCR, MEI, Eduqas, CIE)

$X \sim B(20, p)$. H_0: $p = 0.3$, H_1: $p < 0.3$.
Find the critical region for a test at the 5% significance level.

$P(X \leq 2) = 0.0355 \leq 0.05$
$P(X \leq 3) = 0.107 > 0.05$
Answer: $X \leq 2$

10 Straight line from logarithmic graph (All boards)

Variables x and y satisfy $y = ab^x$. A graph of $\log_{10} y$ against x is a straight line with gradient 0.5 and intercept 1.5.
Find a and b to 3 significant figures.

$\log y = \log a + x \log b$
$\log b = 0.5$ so $b = 10^{0.5} = 3.16$
$\log a = 1.5$ so $a = 10^{1.5} = 31.6$
Answer: a = 31.6, b = 3.16