Day 315 **A Level Maths: C Tier 10-a-day: Model Solutions**

1 Parametric equations: finding points (Edexcel, OCR, MEI, CIE, AQA, Eduqas)

A curve has parametric equations

$x = 4t - 1$, $y = 3t^2 - 3$.

Find the coordinates of the point where $t = -2$.

$x = 4 \times (-2) - 1 = -9$

$y = 3 \times (-2)^2 - 3 = 9$

Answer: $(-9, 9)$

2 Differentiate e^x, ln x, sin and cos (All boards)

Differentiate with respect to x:

(a) $2 \cos 4x$

(b) $2\,e^{4x}$

(a) $dy/dx = -8 \sin 4x$

(b) $dy/dx = 8\,e^{4x}$

Answer: (a) $-8 \sin 4x$ (b) $8\,e^{4x}$

3 Completing the square (AQA, Edexcel, OCR, MEI, Eduqas, CIE)

Write $3x^2 - 12x + 14$ in the form $a(x + p)^2 + q$.

Take out a factor 3: $3(x^2 - 4x + \frac{14}{3})$

$x - 4 = (x - 2)^2 - 4$

$3[(x - 2)^2 - 4] + 14 = 3(x - 2)^2 + 2$

Answer: $3(x - 2)^2 + 2$

4 Velocity-time graph: acceleration from two stages (AQA, Edexcel, OCR, MEI, Eduqas, CIE)

The graph shows the velocity of a skater.

(a) Find the acceleration in the first 2 s.

(b) Find the deceleration in the last 2 s.

(c) Find the total distance travelled.

(a) $a = \frac{8 - 2}{2}$

(b) $a = \frac{8 - 0}{2}$, a deceleration of 4

(c) area $= \frac{1}{2}(2 + 8) \times 2 + 8 \times 6 + \frac{1}{2} \times 2 \times 8$

Answer: (a) 3 m s^{-2} (b) 4 m s^{-2} (c) 66 m

5 Trapezium rule: strips shown (diagram) (AQA, Edexcel, OCR, MEI, Eduqas)

The diagram shows the curve $y = e^x$ from $x = 0$ to $x = 3$ divided into 3 strips of equal width.

Use the trapezium rule with these strips to estimate the shaded area to 3 s.f.

h = 1; y values: 1.000, 2.718, 7.389, 20.09

Area $\approx \frac{1}{2}[1.000 + 20.09 + 2(10.11)]$

$= 20.7$

Answer: 20.7

6 Binomial: cumulative probabilities (AQA, Edexcel, OCR, MEI, Eduqas, CIE)

$X \sim B(19, 0.4)$. Find $P(8 \leq X \leq 11)$, giving your answer to 3 s.f.

$P(8 \leq X \leq 11) = P(X \leq 11) - P(X \leq 7)$

$= 0.9648 - 0.4878$

Answer: 0.477

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7 Sigma notation (AQA, Edexcel, OCR, MEI, Eduqas, CIE)

Evaluate $\sum_{k=1}^{4} 3 \times 3^{k-1}$.

Geometric series: first term 3, common ratio 3, 4 terms.

$S_4 = \frac{3(3^4 - 1)}{3 - 1}$

$= 120$

Answer: 120

8 Implicit differentiation (AQA, Edexcel, OCR, MEI, Eduqas, CIE)

A curve has equation $x^2 + xy = 8$.
Find dy/dx in terms of x and y, and hence the gradient at (2, 2).

Differentiate both sides with respect to x (using dy/dx for y terms)

$dy/dx = -\frac{2x + y}{x}$

At (2, 2): $dy/dx = -3$

Answer: $dy/dx = -\frac{2x + y}{x}$; gradient -3

9 Free fall: dropped from a height (AQA, Edexcel, OCR, MEI, Eduqas, CIE)

A ball is dropped from rest from the top of a cliff of height 78.4 m. Take $g = 9.8$ m s^{-2} and ignore air resistance. Find (a) the time taken to reach the ground, (b) its speed on hitting the ground.

(a) $s = ut + \frac{1}{2}at^2$: $78.4 = \frac{1}{2} \times 9.8 \times t^2$

$78.4 = 4.9t^2$, so $t^2 = 16$

(b) $v = u + at = 9.8 \times 4$

Answer: (a) 4 s (b) 39.2 m s^{-1}

10 Combined mean (AQA, Edexcel, OCR, MEI, Eduqas, CIE)

The mean of 15 ages of the men is 63. The mean of 10 ages of the women is 48.
Find the mean of all 25 values.

Total of first group = $15 \times 63 = 945$
Total of second group = $10 \times 48 = 480$
Mean = $1425 \div 25$
Answer: 57