Day 314 **A Level Maths: C Tier 10-a-day: Model Solutions**

1 Constant acceleration: time to reach a speed (AQA, Edexcel, OCR, MEI, Eduqas, CIE)

A cyclist moving in a straight line increases its speed from 3 m s^{-1} to 36 m s^{-1} with constant acceleration 3 m s^{-2}. Find the time taken.

$v = u + at$
$36 = 3 + 3t$
Answer: 11 s

2 Centre and radius of a circle (AQA, Edexcel, OCR, MEI, Eduqas, CIE)

A circle has equation $x^2 + y^2 + 6x + 8y - 11 = 0$.
Find the centre and the radius of the circle.

Complete the square in x and in y:
$(x + 3)^2 + (y + 4)^2 = 36$
Centre $(-3, -4)$, radius = 6
Answer: Centre $(-3, -4)$, radius 6

3 Integrate (ax + b)^n and related (All boards)

Find $\int \sin(3x + 2)\,dx$.

$\int \sin(ax + b)\,dx = -\frac{1}{a}\cos(ax + b)$

Here a = 3

Answer: $-\frac{1}{3}\cos(3x + 2) + c$

4 Transforming a graph with a labelled turning point (AQA, Edexcel, OCR, MEI, Eduqas, CIE)

The diagram shows the curve y = f(x). The maximum point is shown. Write down the coordinates of the maximum point on the curve with equation
(a) y = f(2x) (b) y = −f(x)

(a) y = f(2x): the maximum point moves to (1, 5).
(b) y = −f(x): the point moves (it becomes a minimum) to (2, −5).
Answer: (a) (1, 5) (b) (2, −5)

5 Resolving a force into components (All boards)

A string pulls a box with a force of 20 N at an angle of 30° above the horizontal. Find the horizontal and vertical components of the force, to 3 s.f.

Horizontal = 20 cos 30°
Vertical = 20 sin 30°
Answer: horizontal 17.3 N, vertical 10.00 N

6 Conditional probability (Bayes) (AQA, Edexcel, OCR, MEI, Eduqas, CIE)

The probability that a patient takes the drug is 0.6. If so, the probability that the patient recovers is 0.5; otherwise it is 0.1. Given that the patient recovers, find the probability that a patient takes the drug.

P(both) = 0.6 × 0.5 = 0.3; P(not first and second) = 0.4 × 0.1 = 0.04
P(second) = 0.3 + 0.04 = 0.34
P = 0.3 ÷ 0.34

Answer: $\frac{15}{17}$ = 0.882

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7 Vector equation of a line (3D) (CIE)

A line has equation $\mathbf{r} = \begin{pmatrix} 3 \\ -3 \\ 1 \end{pmatrix} + t\begin{pmatrix} 2 \\ 1 \\ 2 \end{pmatrix}$.

Find the coordinates of the point on the line where $t = -1$.

$$\mathbf{r} = \begin{pmatrix} 3 \\ -3 \\ 1 \end{pmatrix} + (-1)\begin{pmatrix} 2 \\ 1 \\ 2 \end{pmatrix}$$

$$= \begin{pmatrix} 1 \\ -4 \\ -1 \end{pmatrix}$$

Answer: (1, −4, −1)

8 Normal approximation to the binomial (CIE)

$X \sim B(100, 0.4)$. Use a normal approximation to find $P(X \leq 40)$, to 3 s.f.

$np = 40$ and $npq = 24$, so use $Y \sim N(40, 24)$

Continuity correction: $P(Y < 40.5)$

$z = (40.5 - 40) \div \sqrt{24}$

Answer: 0.541

9 Arithmetic sequences: nth term (AQA, Edexcel, OCR, MEI, Eduqas, CIE)

An arithmetic sequence begins 7, 10, 13, 16, ...

Find (a) an expression for the nth term,

(b) the 15th term.

$a = 7$, $d = 3$

$u_n = a + (n - 1)d = 7 + (n - 1)(3) = 3n + 4$

$u_{15} = 49$

Answer: (a) 3n + 4 (b) 49

10 Newton-Raphson: one or two steps (AQA, Edexcel, OCR, MEI, Eduqas)

$f(x) = \ln x + x - 2$.

Use the Newton-Raphson method with $x_0 = 3$ to find x_1,

giving your answer to 3 decimal places.

$$f'(x) = \frac{1}{x} + 1$$

$$x_{n+1} = x_n - \frac{f(x_n)}{f'(x_n)}$$

$$x_1 = 3 - \frac{2.099}{1.333} = 1.4260$$

Answer: $x_1 = 1.426$