Day 313 **A Level Maths: C Tier 10-a-day: Model Solutions**

1 Factorised trig equation (All boards)

Solve $(\cos x + 1)(2\cos x - 1) = 0$ for $0° \leq x \leq 360°$.

Either $\cos x + 1 = 0$ or $2\cos x - 1 = 0$

$\cos x = -1$: $x = 180°$

$\cos x = \frac{1}{2}$: $x = 60°, 300°$

Answer: $x = 60°, 180°, 300°$

2 Completing the square (AQA, Edexcel, OCR, MEI, Eduqas, CIE)

Write $-2x^2 + 8x - 9$ in the form $a(x + p)^2 + q$.

Take out a factor -2: $-2(x^2 - 4x + \frac{9}{2})$

$x - 4 = (x - 2)^2 - 4$

$-2[(x - 2)^2 - 4] - 9 = -2(x - 2)^2 - 1$

Answer: $-2(x - 2)^2 - 1$

3 Counting: arrangements and selections (CIE)

5 runners take part in a race. In how many ways can the first 4 places be filled?

${}^5P_4 = 5\times4\times3\times2$

Answer: 120

4 Equilibrium of forces given as vectors (AQA, Edexcel, OCR, MEI, Eduqas)

Three forces $(-\mathbf{i} + \mathbf{j})$ N, $(6\mathbf{i} - \mathbf{j})$ N and $(p\mathbf{i} + q\mathbf{j})$ N act on a particle which is in equilibrium. $\mathbf{i}$ and $\mathbf{j}$ are perpendicular unit vectors. Find the values of p and q.

Sum of forces = 0:

i components: $p + (-1) + (6) = 0$

j components: $q + (1) + (-1) = 0$

Answer: $p = -5$, $q = 0$

5 Definite integrals (exact) (All boards)

Find the exact value of

$\int_0^{\frac{\pi}{2}} 10\sin 2x\,dx$.

$[-5\cos 2x]$ from 0 to $\frac{\pi}{2}$

$= -5\cos\pi - (-5\cos 0) = 5 + 5$

Answer: 10

6 Proof by contradiction (AQA, Edexcel, OCR, MEI, Eduqas)

Prove by contradiction that if n^2 is even, then n is even.

Begin: "Assume n is odd, so $n = 2k + 1$ for an integer k."

$n^2 = (2k + 1)^2 = 4k^2 + 4k + 1$

$= 2(2k^2 + 2k + 0) + 1$

This is odd, which contradicts n^2 being even.

So the assumption is false: n must be even.

Answer: Proved

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7 Hypothesis test for a normal mean (AQA, Edexcel, OCR, MEI, Eduqas, CIE)

A population is $N(\mu, 4^2)$. H_0: $\mu = 155$, H_1: $\mu > 155$.

A sample of 9 has mean $\bar{x} = 158.6$.
Test at the 5% level. Critical value: $z = 1.645$.

$z = (\bar{x} - \mu) \div (\sigma \div \sqrt{n}) = (158.6 - 155) \div (4 \div 3)$
$z = 2.70$
2.70 is inside the critical region, so reject H_0
Answer: $z = 2.70$; reject H_0

8 Projectile launched with velocity components (AQA, Edexcel, OCR, MEI)

A stone is projected from level ground with velocity $(11\mathbf{i} + 19.6\mathbf{j})$ m s^{-1} ($\mathbf{i}$ horizontal, $\mathbf{j}$ vertically up). Take $g = 9.8$ m s^{-2}.
(a) Find the time of flight.
(b) Find the horizontal range.

(a) Vertically: $0 = 19.6\,t - 4.9t^2$
$t = \frac{19.6}{4.9} = 4$
(b) Range $= 11 \times 4$
Answer: (a) 4 s (b) 44 m

9 Simplifying algebraic indices (AQA, Edexcel, OCR, MEI, Eduqas, CIE)

Simplify $(3x^4)^2 \div 9x$.

$(3x^4)^2 = 9x^8$
$9x^8 \div 9x$: divide the numbers and subtract the powers
$= x^7$
Answer: x^7

10 Small angle approximations (AQA, Edexcel, OCR, MEI, Eduqas)

θ is small and measured in radians.
Use the small angle approximations to find an approximation for $\cos\theta + \theta \sin 4\theta$.

$\cos\theta \approx 1 - \frac{\theta^2}{2}$, $\theta \sin 4\theta \approx 4\theta^2$

So $\cos\theta + \theta \sin 4\theta \approx 1 + \frac{7\,\theta^2}{2}$

Answer: $1 + \frac{7\,\theta^2}{2}$