Day 312 **A Level Maths: C Tier 10-a-day: Model Solutions**

1 Differentiate e^x, ln x, sin and cos (All boards)

Differentiate with respect to x:
(a) 4 cos 2x
(b) 4 sin 2x

(a) dy/dx = −8 sin 2x
(b) dy/dx = 8 cos 2x
Answer: (a) −8 sin 2x (b) 8 cos 2x

2 Equilibrium of forces given as vectors (AQA, Edexcel, OCR, MEI, Eduqas)

Three forces (p**i** + q**j**) N, (−4**i** + 5**j**) N and (2**i** + **j**) N act on a particle which is in equilibrium. **i** and **j** are perpendicular unit vectors. Find the values of p and q.

Sum of forces = 0:
i components: p + (−4) + (2) = 0
j components: q + (5) + (1) = 0
Answer: p = 2, q = −6

3 Binomial expansion: positive integer powers (AQA, Edexcel, OCR, MEI, Eduqas, CIE)

Find the coefficient of x^2 in the expansion of
$(3 - 2x)^6$.

Term in x^2: ${}^6C_2 \times 3^4 \times (-2x)^2$
$= 15 \times 81 \times 4$
Answer: 4860

4 Conditional probability: two-way tables (AQA, Edexcel, OCR, MEI, Eduqas, CIE)

Of 130 people, 44 are women. 21 of the women and 48 of the men own a bike.
One person is chosen at random.
Given that the person is a woman, find the probability that they own a bike.

There are 44 women
P = 21 ÷ 44
Answer: $\frac{21}{44}$

5 Stone thrown down from a cliff (AQA, Edexcel, OCR, MEI, Eduqas, CIE)

A stone is thrown vertically downwards at 4 m s^{-1} from the top of a cliff and hits the sea after 2 s. Taking g = 9.8 m s^{-2}, find the height of the cliff.

$s = ut + \frac{1}{2}at^2$

$s = 4 \times 2 + \frac{1}{2} \times 9.8 \times 2^2$

$s = 8 + 19.6$
Answer: 27.6 m

6 Addition formulae: exact values (All boards)

By writing 15° as 45° − 30°, find the exact value of sin 15°.

$\sin(45° - 30°) = \sin 45° \cos 30° - \cos 45° \sin 30°$

$= \frac{\sqrt{2}}{2} \times \frac{\sqrt{3}}{2} - \frac{\sqrt{2}}{2} \times \frac{1}{2}$

Simplify: $\frac{\sqrt{6} - \sqrt{2}}{4}$

Answer: $\frac{\sqrt{6} - \sqrt{2}}{4}$

Day 312 **A Level Maths: C Tier 10-a-day: Model Solutions**

7 Equation of a reciprocal graph from asymptotes (AQA, Edexcel, OCR, MEI, Eduqas, CIE)

The diagram shows the graph of $y = \frac{k}{x - a} + c$ with its asymptotes. Find the values of a, c and k.

The asymptotes are $x = -1$ and $y = 1$, so $a = -1$ and $c = 1$.

Through $(0, -1)$: $-1 = \frac{k}{0 - (-1)} + 1$

$\frac{k}{1} = -2$, so $k = -2$

Answer: $a = -1$, $c = 1$, $k = -2$

8 Stationary points of a cubic (All boards)

$y = x^3 - 6x^2 + 1$.

Find the coordinates of the stationary points and determine their nature.

$dy/dx = 3x^2 - 12x = 0$

$x = 0$: $y = 1$; $x = 4$: $y = -31$

$d^2y/dx^2 = 6x - 12$: at $x = 0$ it is -12 (maximum), at $x = 4$ it is 12 (minimum)

Answer: (0, 1) maximum; (4, −31) minimum

9 Tangent condition for a line and a curve (AQA, Edexcel, OCR, MEI, Eduqas, CIE)

The line $y = -x + k$ is a tangent to the curve

$y = x^2 - 3x + 4$.

Find the value of k.

$x^2 - 3x + 4 = -x + k$

$x^2 - 2x + (4 - k) = 0$

A tangent touches once: discriminant = 0

$(-2)^2 - 4(4 - k) = 0$

Answer: $k = 3$

10 Normal distribution: symmetry (AQA, Edexcel, OCR, MEI, Eduqas, CIE)

$X \sim N(110, 3^2)$ and $P(110 < X < 116) = 0.09$.

Find $P(X > 116)$.

$P(X > 110) = 0.5$ as the mean is 110

$P(X > 116) = 0.5 - 0.09$

Answer: 0.41