Day 310 **A Level Maths: C Tier 10-a-day: Model Solutions**

1 Amplitude and period from a graph (diagram) (All boards)

The diagram shows the graph of $y = a \sin bx$ for $0° \leq x \leq 360°$, where a and b are positive integers.
Find a and b.

The maximum value of y is 4, so a = 4

The graph completes 4 full cycles in 360°, so the period is 90° and $b = \frac{360}{90} = 4$

Answer: a = 4, b = 4

2 Expanding brackets with surds (AQA, Edexcel, OCR, MEI, Eduqas, CIE)

Expand and simplify $(7 + \sqrt{7})(7 - \sqrt{7})$.

$= 7^2 - (\sqrt{7})^2$
$= 49 - 7$
Answer: 42

3 Friction: constant speed, find the coefficient (All boards)

A crate of mass 15 kg is pulled along rough horizontal ground at constant speed by a horizontal force of 36.75 N. Take g = 9.8 m s^{-2}. Find the coefficient of friction between the crate and ground.

R = 15 × 9.8 = 147
Constant speed: F = P = 36.75
F = μR: 36.75 = μ × 147
Answer: 0.25

4 Integration by substitution (All boards)

Use the substitution $u = x^2 + 1$ to find

$\int \frac{3x}{x^2 + 1}\, dx$.

$u = x^2 + 1$, $du = 2x\, dx$, so $x\, dx = \frac{1}{2}\, du$

$\int \frac{3}{2u}\, du = \frac{3}{2} \ln |u|$

Answer: $\frac{3}{2} \ln (x^2 + 1) + c$

5 Normal distribution: inverse (AQA, Edexcel, OCR, MEI, Eduqas, CIE)

$X \sim N(115, 5^2)$. Given that $P(X < a) = 0.1$, find a to 3 s.f.

$P(Z < z) = 0.1$, so $z = -1.282$ (from the inverse normal function)
$a = 115 + -1.282 \times 5$
Answer: 109

6 Gradient of a tangent to a circle (AQA, Edexcel, OCR, MEI, Eduqas, CIE)

The circle $(x + 4)^2 + (y + 4)^2 = 13$ passes through P(−6, −7).
Find the gradient of the tangent to the circle at P.

Centre C(−4, −4).

Gradient of $CP = \frac{-7 - (-4)}{-6 - (-4)} = \frac{3}{2}$

The tangent is perpendicular to CP: gradient $= -1 \div \frac{3}{2}$

Answer: $-\frac{2}{3}$

7 Geometric distribution (CIE)

Seeds are planted one at a time until one grows, with probability 0.3 each time. X is the number planted.
Find $P(X \leq 4)$, to 3 s.f.

$P(X \leq 4) = 1 - P(X > 4) = 1 - (\frac{7}{10})^4$

Answer: 0.760

8 Vector geometry with a diagram (OAB) (All boards)

In the diagram, OA = $\mathbf{a}$, OB = $\mathbf{b}$, and P lies on AB with AP : PB = 2 : 3.
Find AB and AP in terms of $\mathbf{a}$ and $\mathbf{b}$.

AB = OB − OA = $\mathbf{b} - \mathbf{a}$

AP = $\frac{2}{5}$ AB = $\frac{2}{5}(\mathbf{b} - \mathbf{a})$

Answer: AB = $\mathbf{b} - \mathbf{a}$; AP = $\frac{2}{5}(\mathbf{b} - \mathbf{a})$

9 Geometric series: sum to infinity (AQA, Edexcel, OCR, MEI, Eduqas, CIE)

A geometric series has common ratio $\frac{3}{4}$ and sum to infinity 40. Find the first term.

$S_\infty = \frac{a}{1 - r}$, so $40 = \frac{a}{1 - \frac{3}{4}}$

$a = 40 \times \frac{1}{4}$

Answer: a = 10

10 Vertical motion: greatest height from speed (AQA, Edexcel, OCR, MEI, Eduqas, CIE)

A rocket is projected vertically upwards with speed 7 m s^{-1} from ground level. Taking g = 9.8 m s^{-2}, find the maximum height it reaches above the ground.

At the top v = 0: $v^2 = u^2 + 2as$

$0 = 7^2 - 2 \times 9.8 \times s$

$s = \frac{49}{19.6}$

Answer: 2.5 m