Day 31 **A Level Maths: C Tier 10-a-day: Model Solutions**

1 2D vectors: magnitude and direction (All boards)

The vector $p = 3\mathbf{i} - 4\mathbf{j}$.
Find (a) |p|, (b) the angle p makes with the positive x-axis, measured anticlockwise (1 d.p.).

(a) $|p| = \sqrt{3^2 + 4^2} = 5$

(b) Acute angle = $\tan^{-1}(\frac{4}{3}) = 53.1°$

p is in the fourth quadrant, so the angle is 306.9°

Answer: (a) 5 (b) 306.9°

2 Linear inequalities (AQA, Edexcel, OCR, MEI, Eduqas, CIE)

Solve $-2x - 8 \geq 3$.

$-2x \geq 11$

Divide by -2 and reverse the inequality:

$x \leq -\frac{11}{2}$

Answer: $x \leq -\frac{11}{2}$

3 Double angle: find sin 2A and cos 2A (All boards)

A is acute and $\cos A = \frac{3}{5}$.

Find the exact values of sin 2A and cos 2A.

$\sin A = \sqrt{1 - \cos^2 A} = \frac{4}{5}$

$\sin 2A = 2 \sin A \cos A = 2 \times \frac{4}{5} \times \frac{3}{5} = \frac{24}{25}$

$\cos 2A = 2\cos^2 A - 1 = 2 \times \frac{9}{25} - 1 = -\frac{7}{25}$

Answer: $\sin 2A = \frac{24}{25}$, $\cos 2A = -\frac{7}{25}$

4 Binomial: modelling conditions (AQA, Edexcel, OCR, MEI, Eduqas, CIE)

X is the number of red cards when 14 cards are dealt from a pack without replacement.
Can X be modelled by a binomial distribution? Give a reason.

A binomial model needs: a fixed number of trials, two outcomes, constant probability, independent trials.
Here the cards are not replaced, so trials are not independent.

Answer: No, the cards are not replaced, so trials are not independent

5 Rationalising the denominator (AQA, Edexcel, OCR, MEI, Eduqas, CIE)

Write $\frac{4 + \sqrt{2}}{\sqrt{2}}$ in the form $p\sqrt{2} + q$.

$\frac{4 + \sqrt{2}}{\sqrt{2}} = \frac{4}{\sqrt{2}} + \frac{\sqrt{2}}{\sqrt{2}}$

$= 2\sqrt{2} + 1$

Answer: $2\sqrt{2} + 1$

6 Continuous random variable: finding k (CIE)

The continuous random variable X has probability density function $f(x) = kx^2$ for $0 \leq x \leq 2$, and 0 otherwise.
Find the value of k.

$\int$ from 0 to 2 of $k\,x^2\,dx = 1$

$k \times \frac{2^3}{3} = 1$

Answer: $k = \frac{3}{8}$

7 Moments: reactions on a uniform beam (AQA, Edexcel, OCR, MEI)

A uniform beam AB of mass 30 kg and length 5 m rests horizontally on supports at A and B. A load of mass 25 kg is placed on the beam 1.25 m from A. Take g = 9.8 m s^{-2}. Find the reaction at B.

Moments about A:

$R_B \times 5 = 30 \times 9.8 \times 2.5 + 25 \times 9.8 \times 1.25$

$R_B \times 5 = 1041.25$

Answer: 208.25 N

8 Definite integrals (exact) (All boards)

Find the exact value of

$\int_1^7 \frac{5}{x}\,dx$.

[5 ln x] from 1 to 7

$= 5 \ln 7 - 5 \ln 1 = 5 \ln 7$

Answer: 5 ln 7

9 Rough slope: limiting equilibrium or sliding (All boards)

A block is on the point of sliding down a rough plane inclined at α to the horizontal, where $\tan \alpha = \frac{3}{4}$. Find the coefficient of friction.

Perpendicular: $R = mg \cos \alpha$

Parallel (limiting): $F = mg \sin \alpha$

$\mu = \frac{F}{R} = \tan \alpha$

Answer: $\frac{3}{4}$

10 Geometric sequences (AQA, Edexcel, OCR, MEI, Eduqas, CIE)

A geometric sequence begins 405, 135, 45, ...

Find the common ratio and the 8th term.

$r = \frac{135}{405} = \frac{1}{3}$

$u_n = ar^{n-1}$, so $u_8 = 405 \times (\frac{1}{3})^7$

$= \frac{5}{27}$

Answer: $r = \frac{1}{3}$, $u_8 = \frac{5}{27}$