Day 306 **A Level Maths: C Tier 10-a-day: Model Solutions**

1 Differentiate e^x, ln x, sin and cos (All boards)

Differentiate with respect to x:

(a) $4\,e^{3x}$

(b) $4\cos 3x$

(a) $dy/dx = 12\,e^{3x}$

(b) $dy/dx = -12\sin 3x$

Answer: (a) $12\,e^{3x}$ (b) $-12\sin 3x$

2 Gradient and parallel or perpendicular lines (AQA, Edexcel, OCR, MEI, Eduqas, CIE)

Find the equation of the line parallel to
$3x - 2y = 12$ that passes through (5, 5).
Give the answer in the form $y = mx + c$.

Rearrange: $y = \frac{3}{2}x - 6$, so the gradient is $\frac{3}{2}$.

$y - 5 = \frac{3}{2}(x - 5)$

$y = \frac{3}{2}x - \frac{5}{2}$

Answer: $y = \frac{3}{2}x - \frac{5}{2}$

3 Vector equation of a line (3D) (CIE)

A line has equation $\mathbf{r} = \begin{pmatrix} -2 \\ 2 \\ 2 \end{pmatrix} + t\begin{pmatrix} -2 \\ 1 \\ -2 \end{pmatrix}$.

Show that the point $(-8, 5, -4)$ lies on the line.

Equate the first coordinates: $-2 + (-2)\,t = -8$, so $t = 3$
Check the others: $2 + 1 \times 3 = 5$ and $2 + (-2) \times 3 = -4$
All three agree, so the point is on the line
Answer: Shown (t = 3)

4 Histograms (AQA, Edexcel, OCR, MEI, Eduqas, CIE)

The histogram shows the times, t minutes, taken by some people to complete a puzzle.
Find the number of people who took between 25 and 50 minutes.

Frequency = frequency density × class width
$25 \leq t < 35$: $5 \times 10 = 50$
$35 \leq t < 50$: $4 \times 15 = 60$
Total = 50 + 60
Answer: 110

5 Partial fractions (Edexcel, OCR, MEI, CIE, AQA, Eduqas)

Express $\frac{-3x + 18}{(x - 2)(x + 2)}$ in partial fractions.

Write $\frac{-3x + 18}{(x - 2)(x + 2)} = \frac{A}{(x - 2)} + \frac{B}{(x + 2)}$
$-3x + 18 = A(x + 2) + B(x - 2)$
$x = 2$: $12 = A(4)$ so $A = 3$
$x = -2$: $24 = B(-4)$ so $B = -6$
Answer: $\frac{3}{(x - 2)} + \frac{-6}{(x + 2)}$

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6 Mutually exclusive events (AQA, Edexcel, OCR, MEI, Eduqas, CIE)

A and B are mutually exclusive, with P(A) = 0.1 and P(B) = 0.2.
(a) Find P(A ∪ B).
(b) Find the probability that neither A nor B happens.

(a) Mutually exclusive, so P(A ∩ B) = 0
P(A ∪ B) = 0.1 + 0.2 = 0.3
(b) 1 − 0.3 = 0.7
Answer: (a) 0.3 (b) 0.7

7 F = ma: find the acceleration (All boards)

A horizontal force of 27 N acts on a box of mass 3 kg on a smooth horizontal surface.
Find the acceleration.

F = ma
27 = 3 a
Answer: 9 m s^{-2}

8 Integrate using double angle formulae (All boards)

Use a double angle formula to find
$\int \sin^2 4x \, dx$.

$\cos 2\theta = 1 - 2\sin^2\theta$, so $\sin^2 4x = \frac{1}{2}(1 - \cos 8x)$

Integrate term by term

Answer: $\frac{x}{2} - \frac{\sin 8x}{16} + c$

9 Combining two inequalities (AQA, Edexcel, OCR, MEI, Eduqas, CIE)

Find the set of values of x satisfying both
$3x - 1 < -10$ and $x^2 + 3x - 10 < 0$.

$3x - 1 < -10$ gives $x < -3$.
$x^2 + 3x - 10 < 0$: $(x + 5)(x - 2) < 0$, so $-5 < x < 2$.
Both are true where the two ranges overlap.
Answer: $-5 < x < -3$

10 Differentiating displacement to find velocity (AQA, Edexcel, OCR, MEI, Eduqas, CIE)

A body moves in a straight line. Its displacement from O at time t seconds is s metres, where $s = 2t^3 + 4t^2 + 5t$.
Find its velocity when t = 1.

$v = \frac{ds}{dt} = 6t^2 + 8t + 5$

When t = 1: $v = 6 \times 1^2 + 8 \times 1 + 5$
Answer: 19 m s^{-1}