Day 305 **A Level Maths: C Tier 10-a-day: Model Solutions**

1 Iteration $x_{n+1} = g(x_n)$ (All boards)

$x_{n+1} = \ln(x_n + 5) + 1$, with $x_0 = 2$.

Find x_1, x_2 and x_3 to 3 decimal places.

$x_1 = 2.9459$

$x_2 = 3.0727$

$x_3 = 3.0885$

Answer: $x_1 = 2.946$, $x_2 = 3.073$, $x_3 = 3.088$

2 Tension in a string supporting a hanging mass (All boards)

A particle of mass 5 kg hangs from a light string and is at rest. Taking $g = 9.8$ m s^{-2}, find the tension in the string.

No acceleration, so $T = mg$

$T = 5 \times 9.8$

Answer: 49 N

3 Parallel vectors (All boards)

$\mathbf{a} = 3\mathbf{i} + 2\mathbf{j}$ and $\mathbf{b} = \frac{3}{2}\mathbf{i} + k\mathbf{j}$, where k is a constant.

Given that a and b are parallel, find the value of k.

Parallel: $\mathbf{b} = \lambda\mathbf{a}$, so $\frac{3}{2} = \lambda \times 3$, giving $\lambda = \frac{1}{2}$

$k = \lambda \times 2 = 1$

Answer: $k = 1$

4 Integrating acceleration to find velocity (AQA, Edexcel, OCR, MEI, Eduqas, CIE)

A particle moves in a straight line. Its acceleration at time t seconds is $(2t + 3)$ m s^{-2}.

When $t = 0$ its velocity is 4 m s^{-1}.

Find its velocity when $t = 5$.

$v = \int a\,dt = t^2 + 3t + c$

$t = 0$, $v = 4$ gives $c = 4$

When $t = 5$: $v = 25 + 15 + 4$

Answer: 44 m s^{-1}

5 Finding a quadratic from its turning point (AQA, Edexcel, OCR, MEI, Eduqas, CIE)

The diagram shows a sketch of a quadratic curve.

Find its equation in the form $y = a(x - p)^2 + q$.

The turning point is $(-3, -2)$, so $y = a(x + 3)^2 - 2$.

The curve passes through (0, 16):

$16 = a(3)^2 - 2$, so $18 = 9a$

$a = 2$

Answer: $y = 2(x + 3)^2 - 2$

6 Poisson distribution (CIE)

The number of events in one interval has a Poisson distribution with mean 0.5.

Find the probability of exactly 1 in one such interval, to 3 s.f.

$P(X = 1) = e^{-0.5} \times 0.5^1 \div 1!$

Answer: 0.303

7 Domain and range (AQA, Edexcel, OCR, MEI, Eduqas, CIE)

$f(x) = \sqrt{x - 4} - 2$.

State the largest possible domain and the range of f.

$x - 4 \geq 0$, so $x \geq 4$.

$\sqrt{\ldots} \geq 0$, so $f(x) \geq -2$.

Answer: Domain $x \geq 4$, range $f(x) \geq -2$

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8 Normal approximation to the binomial (CIE)

X ~ B(120, 0.4). Use a normal approximation to find P(X ≤ 45), to 3 s.f.

np = 48 and npq = 28.8, so use Y ~ N(48, 28.8)

Continuity correction: P(Y < 45.5)

$z = (45.5 - 48) \div \sqrt{28.8}$

Answer: 0.321

9 Separable differential equations (All boards)

Given that dy/dx = 3xy and y = 3 when x = 0,
find y in terms of x.

$\frac{1}{y}\,dy = 3x\,dx$, so $\ln y = \frac{3}{2}x^2 + c$

x = 0, y = 3: c = ln 3

$y = 3\,e^{\frac{3x^2}{2}}$

Answer: $y = 3\,e^{\frac{3x^2}{2}}$

10 Solving equations with indices (AQA, Edexcel, OCR, MEI, Eduqas, CIE)

Solve $4^x = 512$.

$4 = 2^2$ and $512 = 2^9$

$2^{2x} = 2^9$, so 2x = 9

Answer: $x = \frac{9}{2}$