Day 303 **A Level Maths: C Tier 10-a-day: Model Solutions**

1 Rationalising the denominator (AQA, Edexcel, OCR, MEI, Eduqas, CIE)

Rationalise the denominator of $\frac{2}{3-\sqrt{3}}$.

Write your answer in the form $p + q\sqrt{3}$.

Multiply top and bottom by $3 + \sqrt{3}$:

$$\frac{2(3+\sqrt{3})}{(3)^2-(\sqrt{3})^2} = \frac{2(3+\sqrt{3})}{6}$$

Answer: $1 + \frac{1}{3}\sqrt{3}$

2 Normal distribution: probabilities (AQA, Edexcel, OCR, MEI, Eduqas, CIE)

$X \sim N(40, 10^2)$. Find $P(45 < X < 60)$, giving your answer to 3 s.f.

z-values: 0.5 and 2

P(0.5 < Z < 2) from calculator

Answer: 0.286

3 Chain rule: gradient at a point (All boards)

Find the gradient of the curve $y = (x^2 + 2)^2$
at the point where $x = 1$.

Use the chain rule: $dy/dx = 2(x^2 + 2)^1 \times 2x$

At $x = 1$: $dy/dx = 12$

Answer: 12

4 Velocity-time graph: acceleration and distance (AQA, Edexcel, OCR, MEI, Eduqas, CIE)

The graph shows the velocity of a car over 21 seconds as it moves in a straight line.
(a) Find its acceleration in the first 5 s.
(b) Find the total distance travelled.

(a) acceleration = gradient = $\frac{20}{5}$

(b) distance = area of trapezium

$= \frac{1}{2}(12 + 21) \times 20$

Answer: (a) 4 m s^{-2} (b) 330 m

5 Solve trig equation in radians (calculator) (All boards)

Solve $3 \sin x = 1$ for $0 \leq x \leq 2\pi$.
Give your answers to 3 significant figures.

$\sin x = \frac{1}{3}$

Reference angle = $\sin^{-1}(0.3333) = 0.3398$ radians

sin is positive in the first and second quadrants.

x = 0.340, 2.80

Answer: x = 0.340, 2.80

6 Particle held by a string and a horizontal force (All boards)

A ball of weight 75 N hangs from a light string, pulled aside by a horizontal force P N so that the string makes an angle θ with the vertical, where $\tan\theta = \frac{8}{15}$. Find P and the tension T N.

Vertically: $T \cos\theta = 75$

Horizontally: $T \sin\theta = P$

Dividing: $P = 75 \tan\theta$; $T = \sqrt{75^2 + P^2}$

Answer: P = 40, T = 85

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7 Combining two inequalities (AQA, Edexcel, OCR, MEI, Eduqas, CIE)

Find the set of values of x satisfying both

$3x + 4 > 1$ and $x^2 + 2x - 8 < 0$.

$3x + 4 > 1$ gives $x > -1$.

$x^2 + 2x - 8 < 0$: $(x + 4)(x - 2) < 0$, so $-4 < x < 2$.

Both are true where the two ranges overlap.

Answer: $-1 < x < 2$

8 Normal approximation to the binomial (CIE)

$X \sim B(150, 0.6)$. Use a normal approximation to find $P(X \leq 84)$, to 3 s.f.

$np = 90$ and $npq = 36$, so use $Y \sim N(90, 36)$

Continuity correction: $P(Y < 84.5)$

$z = (84.5 - 90) \div \sqrt{36}$

Answer: 0.180

9 Quadratic in sin or cos (All boards)

Solve $2\cos^2 x + 3\cos x + 1 = 0$ for $0° \leq x \leq 360°$.

Factorise: $(\cos x + 1)(2\cos x + 1) = 0$ (as a quadratic in cos x)

$\cos x = -1$ or $\cos x = -\frac{1}{2}$

$x = 120°, 180°, 240°$

Answer: $x = 120°, 180°, 240°$

10 Equation of a reciprocal graph from asymptotes (AQA, Edexcel, OCR, MEI, Eduqas, CIE)

The diagram shows the graph of $y = \frac{k}{x - a} + c$ with its asymptotes. Find the values of a, c and k.

The asymptotes are $x = -2$ and $y = 2$, so $a = -2$ and $c = 2$.

Through (0, 3): $3 = \frac{k}{0 - (-2)} + 2$

$\frac{k}{2} = 1$, so $k = 2$

Answer: $a = -2$, $c = 2$, $k = 2$