Day 301 **A Level Maths: C Tier 10-a-day: Model Solutions**

1 Modelling assumptions (All boards)

In a model, a beam is modelled as a rod.
State what this assumption means for the motion or forces in the model.

Modelling assumption: beam as a rod.
The assumption means: i.t is one-dimensional: its thickness is ignored
Answer: It is one-dimensional: its thickness is ignored

2 2D vectors: magnitude and direction (All boards)

The vector $\mathbf{p} = 24\mathbf{i} - 7\mathbf{j}$.
Find (a) |p|, (b) the angle p makes with the positive x-axis, measured anticlockwise (1 d.p.).

(a) $|\mathbf{p}| = \sqrt{24^2 + 7^2} = 25$

(b) Acute angle = $\tan^{-1}(\frac{7}{24}) = 16.3°$

p is in the fourth quadrant, so the angle is 343.7°
Answer: (a) 25 (b) 343.7°

3 Centre and radius of a circle (AQA, Edexcel, OCR, MEI, Eduqas, CIE)

A circle has equation $x^2 + y^2 - 12x - 10y + 33 = 0$.
Find the centre and the radius of the circle.

Complete the square in x and in y:
$(x - 6)^2 + (y - 5)^2 = 28$
Centre (6, 5), radius = $2\sqrt{7}$
Answer: Centre (6, 5), radius $2\sqrt{7}$

4 Comparing distributions (AQA, Edexcel, OCR, MEI, Eduqas, CIE)

The heights in cm for two groups: Group A has mean 59, standard deviation 14; Group B has mean 79, standard deviation 5.
Compare the two sets of data.

The mean measures the average: Group B has the higher mean.
The standard deviation measures spread: Group B has the smaller standard deviation, so it is more consistent.
Answer: Group B has the higher mean; Group B is more consistent

5 Inverse functions (AQA, Edexcel, OCR, MEI, Eduqas, CIE)

$f(x) = x^2$, $x \geq 0$.
Find $f^{-1}(x)$ and state its domain.

$y = x^2$, so $x = \sqrt{y}$ (as $x \geq 0$)
The domain of f^{-1} is the range of f: $y \geq 0$
Answer: $f^{-1}(x) = \sqrt{x}$, $x \geq 0$

6 Catching up: constant speed against acceleration (AQA, Edexcel, OCR, MEI, Eduqas, CIE)

Car A passes a point O at constant speed 6 m s^{-1}.
At that instant car B starts from rest at O with constant acceleration 1 m s^{-2} in the same direction. Find the time when B catches up A.

Distances equal: $6t = \frac{1}{2} \times 1 \times t^2$
$t(6 - 0.5t) = 0$, so $t = 0$ or $t = 6 \div 0.5$
Answer: 12 s

Day 301 **A Level Maths: C Tier 10-a-day: Model Solutions**

7 Vector equation of a line (3D) (CIE)

A line has equation $\mathbf{r} = \begin{pmatrix} -1 \\ -2 \\ 1 \end{pmatrix} + t\begin{pmatrix} 2 \\ 1 \\ 1 \end{pmatrix}$.

Show that the point $(-3, -3, 0)$ lies on the line.

Equate the first coordinates: $-1 + 2t = -3$, so $t = -1$
Check the others: $-2 + 1 \times (-1) = -3$ and $1 + 1 \times (-1) = 0$
All three agree, so the point is on the line
Answer: Shown ($t = -1$)

8 Binomial: mean and variance (AQA, Edexcel, OCR, MEI, Eduqas, CIE)

$X \sim B(25, 0.6)$. Find $E(X)$ and $\text{Var}(X)$.

$E(X) = np = 25 \times 0.6 = 15$
$\text{Var}(X) = npq = 25 \times 0.6 \times 0.4$
Answer: $E(X) = 15$, $\text{Var}(X) = 6$

9 Integrate using partial fractions (All boards)

$\frac{-3x - 5}{(x + 1)(x + 3)} \equiv \frac{A}{x + 1} + \frac{B}{x + 3}$.

Find A and B, and hence find $\int \frac{-3x - 5}{(x + 1)(x + 3)}\,dx$.

$-3x - 5 \equiv A(x + 3) + B(x + 1)$
$x = -1$ gives $A = -1$; $x = -3$ gives $B = -2$
$\int \left(\frac{-1}{x + 1} + \frac{-2}{x + 3}\right) dx = -1 \ln|x + 1| + -2 \ln|x + 3| + c$
Answer: $A = -1$, $B = -2$; $-\ln|x + 1| - 2\ln|x + 3| + c$

10 Completing the square (AQA, Edexcel, OCR, MEI, Eduqas, CIE)

Write $3x^2 - 12x + 4$ in the form $a(x + p)^2 + q$.

Take out a factor 3: $3(x^2 - 4x + \frac{4}{3})$

$x - 4 = (x - 2)^2 - 4$
$3[(x - 2)^2 - 4] + 4 = 3(x - 2)^2 - 8$
Answer: $3(x - 2)^2 - 8$