Day 300 **A Level Maths: C Tier 10-a-day: Model Solutions**

1 Intercepts of a cubic graph (AQA, Edexcel, OCR, MEI, Eduqas, CIE)

Sketch $y = (x + 1)(x - 4)(x - 5)$. Write down the coordinates of the points where the curve meets the axes.

$y = 0$: $x = -1, 4, 5$

$x = 0$: $y = 1 \times (-4) \times (-5) = 20$

Answer: $(-1, 0)$, $(4, 0)$, $(5, 0)$, $(0, 20)$

2 Constant acceleration: $v^2 = u^2 + 2as$ (AQA, Edexcel, OCR, MEI, Eduqas, CIE)

A particle moves in a straight line with constant acceleration. Its speed increases from 7 m s^{-1} to 19 m s^{-1} as it travels 312 m. Find the acceleration.

$v^2 = u^2 + 2as$

$19^2 = 7^2 + 2 \times a \times 312$

$312 = 624\,a$

Answer: 0.5 m s^{-2}

3 Trapezium rule from a table (AQA, Edexcel, OCR, MEI, Eduqas)

The table shows values of $y = f(x)$.

x	1	1.25	1.5	1.75	2
y	6.1	2.1	1.5	6.9	8.5

Use the trapezium rule to estimate the area under the curve between $x = 1$ and $x = 2$.

$h = 0.25$ with 4 strips

$\text{Area} \approx \frac{0.25}{2}[6.1 + 8.5 + 2(2.1 + 1.5 + 6.9)]$

$= \frac{0.25}{2} \times 35.6$

$= 4.45$

Answer: 4.45

4 Modulus graphs (AQA, Edexcel, OCR, MEI, Eduqas, CIE)

Sketch $y = |4x + 3|$. Write down the coordinates of the vertex and of the point where the graph meets the y-axis.

The vertex is on the x-axis where $4x + 3 = 0$: $x = -\frac{3}{4}$

$x = 0$: $y = |3| = 3$

Answer: Vertex $(-\frac{3}{4}, 0)$; y-axis $(0, 3)$

5 Inclined plane: normal reaction and holding force (All boards)

A particle of mass 13 kg rests on a smooth plane inclined at α to the horizontal, where $\tan\alpha = \frac{4}{3}$.

It is held by a force P N up the slope. Take $g = 9.8$ m s^{-2}. Find P and the normal reaction.

Parallel: $P = mg\sin\alpha = 13 \times 9.8 \times \frac{4}{5}$

Perpendicular: $R = mg\cos\alpha = 13 \times 9.8 \times \frac{3}{5}$

Answer: P = 101.92, R = 76.44

6 Area between a line and a curve (All boards)

The curve $y = 6x - x^2$ and the line $y = 1x$ meet at the origin and at P.

Find the area enclosed between them.

$6x - x^2 = 1x$ gives $5x - x^2 = 0$, so $x = 0$ or $x = 5$

$\text{Area} = \int_0^5 (5x - x^2)\,dx = [\frac{5x^2}{2} - \frac{x^3}{3}]$ from 0 to 5

$= \frac{5^3}{2} - \frac{5^3}{3} = \frac{125}{6}$

Answer: $\frac{125}{6}$

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7 Critical region (AQA, Edexcel, OCR, MEI, Eduqas, CIE)

$X \sim B(40, p)$. H_0: $p = 0.25$, H_1: $p > 0.25$.

Find the critical region for a test at the 1% significance level.

$P(X \geq 18) = 0.00465 \leq 0.01$

$P(X \geq 17) = 0.0116 > 0.01$

Answer: $X \geq 18$

8 Binomial expansion: non-integer powers (Edexcel, AQA, OCR, MEI, Eduqas, CIE)

Expand $(1 + 2x)^{-1}$ in ascending powers of x up to and including the term in x^2. State the range of values of x for which the expansion is valid.

$(1 + y)^n = 1 + ny + \frac{n(n-1)}{2!}y^2 + \ldots$ with $n = -1$, $y = 2x$

$= 1 + (-1)(2x) + \frac{(-1)((-2))}{2}(2x)^2$

$= 1 - 2x + 4x^2$

Valid when $|2x| < 1$, i.e. $|x| < \frac{1}{2}$

Answer: $1 - 2x + 4x^2$; $|x| < \frac{1}{2}$

9 Discrete random variable: E(X) and Var(X) (AQA, Edexcel, OCR, MEI, Eduqas, CIE)

The discrete random variable X has this distribution.

x: 2, 3, 4, 5

P(X = x): 0.3, 0.1, 0.3, 0.3

Find E(X) and Var(X).

$E(X) = \Sigma xp = 2\times0.3 + 3\times0.1 + 4\times0.3 + 5\times0.3 = 3.6$

$E(X^2) = \Sigma x^2 p = 14.4$

$Var(X) = 14.4 - 3.6^2$

Answer: E(X) = 3.6, Var(X) = 1.44

10 Quadratic in sin or cos (All boards)

Solve $\sin^2 x - \sin x = 0$ for $0° \leq x \leq 360°$.

Factorise: $(\sin x - 1)\sin x = 0$ (as a quadratic in sin x)

$\sin x = 1$ or $\sin x = 0$

$x = 0°, 90°, 180°, 360°$

Answer: x = 0°, 90°, 180°, 360°