Day 30 **A Level Maths: C Tier 10-a-day: Model Solutions**

1 Real roots and the discriminant (AQA, Edexcel, OCR, MEI, Eduqas, CIE)

Find the set of values of k for which
$3x^2 - 2x + k = 0$ has real roots.

$b^2 - 4ac \geq 0$
$4 - 12k \geq 0$
Answer: $k \leq \frac{1}{3}$

2 Median and interquartile range (AQA, Edexcel, OCR, MEI, Eduqas, CIE)

Here are 11 values: 6, 27, 79, 75, 28, 17, 9, 44, 42, 65, 11
Find the median and the interquartile range.

Ordered: 6, 9, 11, 17, 27, 28, 42, 44, 65, 75, 79
Median = 28, lower quartile = 11, upper quartile = 65
IQR = 65 − 11
Answer: median = 28, IQR = 54

3 Area enclosed by a parabola (diagram) (All boards)

The diagram shows the curve $y = 3x(6 - x)$.
Find the area of the shaded region, between the curve and the x-axis.

The curve meets the x-axis where x = 0 and x = 6
Area = $\int_0^6 3(6x - x^2)\,dx = [3(\frac{6x^2}{2} - \frac{x^3}{3})]$ from 0 to 6
$= 3(\frac{6^3}{2} - \frac{6^3}{3})$
$= 108$
Answer: 108

4 Stone thrown down from a cliff (AQA, Edexcel, OCR, MEI, Eduqas, CIE)

A stone is thrown vertically downwards at 12 m s^{-1} from the top of a cliff and hits the sea after 3 s. Taking g = 9.8 m s^{-2}, find the height of the cliff.

$s = ut + \frac{1}{2}at^2$
$s = 12 \times 3 + \frac{1}{2} \times 9.8 \times 3^2$
$s = 36 + 44.1$
Answer: 80.1 m

5 Proof by exhaustion (AQA, Edexcel, OCR, MEI, Eduqas)

Use proof by exhaustion to show that $n^3 - n$ is divisible by 6 for every integer n with $1 \leq n \leq 4$.

Substitute each value of n:
n = 1: 0 = 6 × 0 ; n = 2: 6 = 6 × 1
n = 3: 24 = 6 × 4 ; n = 4: 60 = 6 × 10
Every value is a multiple of 6, so the statement is true for $1 \leq n \leq 4$.
Answer: Shown

6 Constant velocity vector: when does it reach a point? (AQA, Edexcel, OCR, MEI, Eduqas)

A particle moves with constant velocity
$(-2\mathbf{i} + 3\mathbf{j})$ m s^{-1}. It is at the point with position vector $(\mathbf{i} - \mathbf{j})$ m at t = 0. Find the time when its position vector is $(-11\mathbf{i} + 17\mathbf{j})$ m.

$r = r_0 + vt$
$(-11\mathbf{i} + 17\mathbf{j}) = (\mathbf{i} - \mathbf{j}) + t(-2\mathbf{i} + 3\mathbf{j})$
Equating i components: $-11 = 1 + -2t$
Answer: 6 s

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7 Normal approximation to the binomial (CIE)

X ~ B(40, 0.25). Use a normal approximation to find P(X ≤ 9), to 3 s.f.

np = 10 and npq = 7.5, so use Y ~ N(10, 7.5)
Continuity correction: P(Y < 9.5)
$z = (9.5 - 10) \div \sqrt{7.5}$
Answer: 0.428

8 Angle between vectors (scalar product) (CIE)

$\mathbf{a} = -3\mathbf{i} + 4\mathbf{j} + \mathbf{k}$ and $\mathbf{b} = 4\mathbf{i} + \mathbf{j} + 2\mathbf{k}$.
Find the angle between a and b, to 1 decimal place.

$\mathbf{a}.\mathbf{b} = (-3)(4) + (4)(1) + (1)(2) = -6$
$|\mathbf{a}| = \sqrt{26}$, $|\mathbf{b}| = \sqrt{21}$
$\cos\theta = \frac{-6}{\sqrt{26} \times \sqrt{21}}$, so $\theta = 104.9°$
Answer: 104.9°

9 Partial fractions (Edexcel, OCR, MEI, CIE, AQA, Eduqas)

Express $\frac{-x + 7}{(x + 2)(x - 1)}$ in partial fractions.

Write $\frac{-x + 7}{(x + 2)(x - 1)} = \frac{A}{(x + 2)} + \frac{B}{(x - 1)}$
$-x + 7 = A(x - 1) + B(x + 2)$
$x = -2$: $9 = A(-3)$ so $A = -3$
$x = 1$: $6 = B(3)$ so $B = 2$
Answer: $\mathbf{\frac{-3}{(x + 2)} + \frac{2}{(x - 1)}}$

10 Implicit differentiation (AQA, Edexcel, OCR, MEI, Eduqas, CIE)

A curve has equation $2x^2 + 3y^2 = 21$.
Find dy/dx in terms of x and y, and hence the gradient at (3, 1).

Differentiate both sides with respect to x (using dy/dx for y terms)
$\frac{dy}{dx} = -\frac{2x}{3y}$
At (3, 1): $\frac{dy}{dx} = -2$
Answer: $\mathbf{\frac{dy}{dx} = -\frac{2x}{3y}}$**; gradient** $\mathbf{-2}$