Day 3 **A Level Maths: C Tier 10-a-day: Model Solutions**

1 Solve trig equation (calculator, degrees) (All boards)

Solve $\tan x = -3.00$ for $0° \leq x \leq 360°$.
Give your answers to 1 decimal place.

Reference angle (positive value): $\tan^{-1}(3.00) = 71.6°$
tan is negative in the second and fourth quadrants.
$x = 108.4°$ or $x = 288.4°$
Answer: $x = 108.4°, 288.4°$

2 Completing the square (AQA, Edexcel, OCR, MEI, Eduqas, CIE)

Write $-2x^2 - 4x$ in the form $a(x + p)^2 + q$.

Take out a factor -2: $-2(x^2 + 2x)$
$x + 2 = (x + 1)^2 - 1$
$-2[(x + 1)^2 - 1] + 0 = -2(x + 1)^2 + 2$
Answer: $-2(x + 1)^2 + 2$

3 Vertical motion: greatest height from speed (AQA, Edexcel, OCR, MEI, Eduqas, CIE)

A cricket ball is projected vertically upwards with speed 21 m s^{-1} from ground level. Taking g = 9.8 m s^{-2}, find the maximum height it reaches above the ground.

At the top $v = 0$: $v^2 = u^2 + 2as$
$0 = 21^2 - 2 \times 9.8 \times s$
$s = \frac{441}{19.6}$
Answer: 22.5 m

4 Area between a line and a curve (All boards)

The curve $y = 4x - x^2$ and the line $y = 1x$ meet at the origin and at P.
Find the area enclosed between them.

$4x - x^2 = 1x$ gives $3x - x^2 = 0$, so $x = 0$ or $x = 3$
Area $= \int_0^3 (3x - x^2)\,dx = [\frac{3x^2}{2} - \frac{x^3}{3}]$ from 0 to 3
$= \frac{3^3}{2} - \frac{3^3}{3} = \frac{9}{2}$
Answer: $\frac{9}{2}$

5 Equation of a transformed curve (AQA, Edexcel, OCR, MEI, Eduqas, CIE)

The curve $y = x^2 - 4x + 3$ is translated by the vector $\begin{pmatrix}1\\4\end{pmatrix}$.

Find the equation of the new curve, giving your answer in the form $y = ax^2 + bx + c$.

Replace x by $(x - 1)$ and add 4:
$y = f(x - 1) + 4$
Expand and collect terms.
Answer: $y = x^2 - 6x + 12$

6 Normal distribution: symmetry (AQA, Edexcel, OCR, MEI, Eduqas, CIE)

$X \sim N(120, 3^2)$ and $P(120 < X < 126) = 0.075$.
Find $P(X > 126)$.

$P(X > 120) = 0.5$ as the mean is 120
$P(X > 126) = 0.5 - 0.075$
Answer: 0.425

7 Maximum speed from calculus (AQA, Edexcel, OCR, MEI, Eduqas, CIE)

A particle moves in a straight line. Its velocity at time t seconds ($0 \le t \le 2$) is $v = (-4t^2 + 8t)$ m s^{-1}. Find the maximum velocity and the time at which it occurs.

$a = \frac{dv}{dt} = 8 - 8t$

Maximum when $a = 0$: $t = \frac{8}{8} = 1$

$v = 8 \times 1 - 4 \times 1^2$

Answer: 4 m s^{-1} at t = 1

8 Integration by substitution (All boards)

Use the substitution $u = x^2 + 3$ to find

$\int 4x(x^2 + 3)^4 \, dx$.

$u = x^2 + 3$, so $du = 2x \, dx$

$\int 2u^4 \, du = \frac{2u^5}{5}$

Replace u: the answer below

Answer: $\frac{2(x^2 + 3)^5}{5} + c$

9 Conditional probability (Bayes) (AQA, Edexcel, OCR, MEI, Eduqas, CIE)

The probability that a machine is set up wrongly is 0.4. If so, the probability that an item is faulty is 0.6; otherwise it is 0.4. Given that an item is faulty, find the probability that a machine is set up wrongly.

P(both) = 0.4 × 0.6 = 0.24; P(not first and second) = 0.6 × 0.4 = 0.24
P(second) = 0.24 + 0.24 = 0.48
P = 0.24 ÷ 0.48

Answer: $\frac{1}{2} = 0.5$

10 Binomial expansion: non-integer powers (Edexcel, AQA, OCR, MEI, Eduqas, CIE)

Expand $(1 + 3x)^{-1}$ in ascending powers of x up to and including the term in x^2. State the range of values of x for which the expansion is valid.

$(1 + y)^n = 1 + ny + \frac{n(n-1)}{2!}y^2 + \ldots$ with $n = -1$, $y = 3x$

$= 1 + (-1)(3x) + \frac{(-1)((-2))}{2}(3x)^2$

$= 1 - 3x + 9x^2$

Valid when $|3x| < 1$, i.e. $|x| < \frac{1}{3}$

Answer: $1 - 3x + 9x^2$; $|x| < \frac{1}{3}$