Day 299 **A Level Maths: C Tier 10-a-day: Model Solutions**

1 Two objects moving towards each other (AQA, Edexcel, OCR, MEI, Eduqas, CIE)

Two cyclists start at the same time from points P and Q, 33 m apart, and cycle directly towards each other at constant speeds of 7 m s^{-1} and 4 m s^{-1}. Find the time until they meet.

Closing speed = 7 + 4 = 11 m s^{-1}

Time = $\frac{33}{11}$

Answer: 3 s

2 Integrate standard functions (All boards)

Find

(a) $\int \frac{5}{x}\,dx$

(b) $\int 10 \sin 2x\,dx$

Use the standard integrals and divide by the coefficient of x where needed.

(a) = 5 ln |x| + c

(b) = −5 cos 2x + c

Answer: (a) 5 ln |x| + c (b) −5 cos 2x + c

3 Histograms (AQA, Edexcel, OCR, MEI, Eduqas, CIE)

The histogram shows the times, t minutes, taken by some people to complete a puzzle.
Find the number of people who took between 5 and 10 minutes.

Frequency = frequency density × class width

$5 \leq t < 10$: 3 × 5 = 15

Answer: 15

4 Product rule (All boards)

Use the product rule to differentiate

$y = x^2 \sin 3x$.

$u = x^2$, $v = \sin 3x$

$u' = 2x^1$, $v' = 3 \cos 3x$

dy/dx = u′v + uv′, which simplifies to the answer below

Answer: $dy/dx = x^1(2 \sin 3x + 3x \cos 3x)$

5 Linear and quadratic simultaneous equations (AQA, Edexcel, OCR, MEI, Eduqas, CIE)

Solve the simultaneous equations:

$y = x^2 + 2x + 6$, $y = -x + 6$.

$x^2 + 2x + 6 = -x + 6$

$x^2 + 3x = 0$

(x + 3)(x + 0) = 0

Use $y = -x + 6$ to find each y.

Answer: x = −3, y = 9 and x = 0, y = 6

6 Constant acceleration: finding the initial speed (AQA, Edexcel, OCR, MEI, Eduqas, CIE)

A particle moving in a straight line with constant acceleration 1 m s^{-2} travels 57.5 m in 5 s.
Find its initial speed.

$s = ut + \frac{1}{2}at^2$

$57.5 = 5u + \frac{1}{2} \times 1 \times 5^2$

5 u = 45

Answer: 9 m s^{-1}

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7 Equation of a reciprocal graph from asymptotes (AQA, Edexcel, OCR, MEI, Eduqas, CIE)

The diagram shows the graph of $y = \frac{k}{x - a} + c$ with its asymptotes. Find the values of a, c and k.

The asymptotes are $x = 1$ and $y = 2$, so $a = 1$ and $c = 2$.

Through $(0, -1)$: $-1 = \frac{k}{0 - 1} + 2$

$\frac{k}{-1} = -3$, so $k = 3$

Answer: a = 1, c = 2, k = 3

8 Binomial test using a given probability (AQA, Edexcel, OCR, MEI, Eduqas, CIE)

$X \sim B(15, p)$. H_0: $p = 0.2$, H_1: $p < 0.2$.

In a sample, $x = 1$. It is given that $P(X \leq 1) = 0.1671$.

Test H_0 at the 10% significance level.

Compare 0.1671 with 0.1

0.1671 > 0.1, so do not reject H_0

Answer: Do not reject H_0: insufficient evidence that p is less than 0.2

9 Tangent condition for a line and a curve (AQA, Edexcel, OCR, MEI, Eduqas, CIE)

The line $y = -4x + k$ is a tangent to the curve

$y = x^2 + 6$.

Find the value of k.

$x^2 + 6 = -4x + k$

$x^2 + 4x + (6 - k) = 0$

A tangent touches once: discriminant = 0

$(4)^2 - 4(6 - k) = 0$

Answer: k = 2

10 Solve trig equation in radians (calculator) (All boards)

Solve $2 \cos x = -1$ for $0 \leq x \leq 2\pi$.

Give your answers to 3 significant figures.

$\cos x = -\frac{1}{2}$

Reference angle = $\cos^{-1}(0.5000) = 1.047$ radians

cos is negative in the second and third quadrants.

$x = 2.09, 4.19$

Answer: x = 2.09, 4.19