Day 297 **A Level Maths: C Tier 10-a-day: Model Solutions**

1 Resolving a force into components (All boards)

A cable pulls a box with a force of 30 N at an angle of 60° above the horizontal. Find the horizontal and vertical components of the force, to 3 s.f.

Horizontal = 30 cos 60°
Vertical = 30 sin 60°
Answer: horizontal 15.0 N, vertical 26.0 N

2 Selection without replacement (AQA, Edexcel, OCR, MEI, Eduqas, CIE)

A bag has 3 red and 5 blue sweets. Two are taken at random without replacement.
Find the probability that they are different colours.

P(RB) = $\frac{3}{8} \times \frac{5}{7}$, P(BR) = $\frac{5}{8} \times \frac{3}{7}$

Add: $\frac{15}{56} + \frac{15}{56}$

Answer: $\frac{15}{28}$

3 Completing the square (AQA, Edexcel, OCR, MEI, Eduqas, CIE)

Write $-2x^2 - 12x - 20$ in the form $a(x + p)^2 + q$.

Take out a factor -2: $-2(x^2 + 6x + 10)$
$x + 6 = (x + 3)^2 - 9$
$-2[(x + 3)^2 - 9] - 20 = -2(x + 3)^2 - 2$
Answer: $-2(x + 3)^2 - 2$

4 Outliers (Edexcel, OCR, MEI)

For a data set $Q_1 = 28$ and $Q_3 = 58$.
A value is an outlier if it is more than 1.5 × IQR above Q_3 or below Q_1.
Determine whether -12 and 33 are outliers.

IQR = 58 − 28 = 30, so 1.5 × IQR = 45
Lower limit = 28 − 45 = −17; upper limit = 58 + 45 = 103
−12 is not an outlier; 33 is not an outlier
Answer: −12 is not an outlier; 33 is not an outlier

5 Equation of a tangent or normal (All boards)

Find the equation of the tangent to the curve
$y = e^{3x} + x$ at the point where $x = 0$.

dy/dx = 3exp(3x) + 1, so the gradient of the curve at x = 0 is 4
At x = 0, y = 1
y − 1 = 4(x − 0)
Answer: y = 4x + 1

6 Real roots and the discriminant (AQA, Edexcel, OCR, MEI, Eduqas, CIE)

Find the set of values of k for which
$x^2 + kx + 16 = 0$ has two distinct real roots.

$b^2 - 4ac > 0$
$k^2 - 64 > 0$, so $k^2 > 64$
Answer: $k < -8$ or $k > 8$

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7 Rearrange and iterate (All boards)

(a) Show that $x^2 - x - 2 = 0$ can be rearranged to $x = 1 + \frac{2}{x}$.

(b) Use x_{n+1} = this formula with $x_0 = 2$ to find x_1, x_2 to 3 d.p.

(a) Divide $x^2 - x = 2$ by x: $x - 1 = \frac{2}{x}$, so $x = 1 + \frac{2}{x}$

(b) $x_1 = 2.0000$, $x_2 = 2.0000$

Answer: (a) shown (b) $x_1 = 2.000$, $x_2 = 2.000$

8 Simplifying surds (AQA, Edexcel, OCR, MEI, Eduqas, CIE)

Write $\sqrt{150}$ in the form $p\sqrt{q}$, where q is an integer as small as possible.

$\sqrt{150} = \sqrt{25 \times 6} = \sqrt{25} \times \sqrt{6}$

$= 5\sqrt{6}$

Answer: $5\sqrt{6}$

9 Maximum value from R form (All boards)

f(x) = 6 sin x + 8 cos x + 4.

Write f(x) in the form R sin(x + α) + 4, with α in degrees to 1 d.p., and state the maximum value of f(x).

$R = \sqrt{6^2 + 8^2} = 10$, $\tan \alpha = \frac{8}{6}$ so $\alpha = 53.1°$

f(x) = 10 sin(x + 53.1°) + 4

sin(x + α) has maximum value 1, so f(max) = 10 + 4

Answer: f(x) = 10 sin(x + 53.1°) + 4; maximum 14

10 Two-stage motion: acceleration then constant speed (AQA, Edexcel, OCR, MEI, Eduqas, CIE)

A train starts from rest and accelerates uniformly at 4 m s^{-2} for 6 s, then travels at constant speed for a further 30 s. Find the total distance travelled.

Speed reached: $v = 4 \times 6 = 24$ m s^{-1}

Stage 1: $s = \frac{1}{2} \times 24 \times 6 = 72$

Stage 2: $s = 24 \times 30 = 720$

Answer: 792 m