Day 296 **A Level Maths: C Tier 10-a-day: Model Solutions**

1 Newton-Raphson: one or two steps (AQA, Edexcel, OCR, MEI, Eduqas)

$f(x) = x^3 - 2x - 6$.

Use the Newton-Raphson method with $x_0 = 3$ to find x_1 and x_2, giving your answer to 3 decimal places.

$f'(x) = 3x^2 - 2$

$x_{n+1} = x_n - \frac{f(x_n)}{f'(x_n)}$

$x_1 = 3 - \frac{15.00}{25.00} = 2.4000$

$x_2 = 2.4000 - \frac{3.024}{15.28} = 2.2021$

Answer: $x_1 = 2.400$, $x_2 = 2.202$

2 Midpoint and length of a line segment (AQA, Edexcel, OCR, MEI, Eduqas, CIE)

A is $(-4, 4)$ and B is $(-4, -4)$.

Find (a) the midpoint of AB,

(b) the length of AB, as a surd in simplest form.

(a) Midpoint $= (\frac{-4 + (-4)}{2}, \frac{4 + (-4)}{2}) = (-4, 0)$

(b) $AB^2 = (0)^2 + (-8)^2 = 0 + 64 = 64$

$AB = \sqrt{64} = 8$

Answer: (a) $(-4, 0)$ (b) 8

3 Inverse trig functions (Edexcel, OCR)

Find the exact value, in radians, of

(a) $\arccos(-\frac{1}{2})$ (b) $\arccos(1)$

(a) $y = \arccos(-\frac{1}{2})$ means $\cos y = -\frac{1}{2}$ with $0 \leq y \leq \pi$: $y = \frac{2\pi}{3}$

(b) $y = \arccos(1)$ means $\cos y = 1$ with $0 \leq y \leq \pi$: $y = 0$

Answer: (a) $\frac{2\pi}{3}$ (b) 0

4 Critical region (AQA, Edexcel, OCR, MEI, Eduqas, CIE)

$X \sim B(40, p)$. H_0: $p = 0.6$, H_1: $p < 0.6$.

Find the critical region for a test at the 1% significance level.

$P(X \leq 16) = 0.00834 \leq 0.01$

$P(X \leq 17) = 0.0189 > 0.01$

Answer: $X \leq 16$

5 Friction: acceleration along rough ground (All boards)

A block of mass 2 kg is pulled along rough horizontal ground by a horizontal force of 13.8 N. The coefficient of friction is 0.5. Take $g = 9.8$ m s^{-2}. Find the acceleration.

$R = 2 \times 9.8 = 19.6$

$F = \mu R = 0.5 \times 19.6 = 9.8$

$P - F = ma$: $13.8 - 9.8 = 2a$

Answer: 2 m s^{-2}

6 Solving equations with indices (AQA, Edexcel, OCR, MEI, Eduqas, CIE)

Solve $2^{4x} = 64$.

$64 = 2^6$, so $4x = 6$

$4x = 6$

Answer: $x = \frac{3}{2}$

7 Inclined plane: normal reaction and holding force (All boards)

A particle of mass 5 kg rests on a smooth plane inclined at α to the horizontal, where $\tan\alpha = \frac{4}{3}$.

It is held by a force P N up the slope. Take g = 9.8 m s^{-2}. Find P and the normal reaction.

Parallel: $P = mg\sin\alpha = 5 \times 9.8 \times \frac{4}{5}$

Perpendicular: $R = mg\cos\alpha = 5 \times 9.8 \times \frac{3}{5}$

Answer: P = 39.2, R = 29.4

8 Solve with a shifted angle (All boards)

Solve $\cos(x + 30°) = \frac{\sqrt{2}}{2}$ for $0° \leq x \leq 360°$.

Let $u = x + 30°$, so $30° \leq u \leq 390°$

$\cos u = \frac{\sqrt{2}}{2}$: $u = 45°, 315°$

$x = u - 30°$: $x = 15°, 285°$

Answer: x = 15°, 285°

9 Discrete random variable: finding k (AQA, Edexcel, OCR, MEI, Eduqas, CIE)

The discrete random variable X has $P(X = x) = k(7 - x)$ for x = 1, 2, 3, 4, 5.

(a) Find the value of k.

(b) Find $P(X \geq 2)$.

(a) Probabilities sum to 1: $k(6 + 5 + 4 + 3 + 2) = 1$

$k = \frac{1}{20}$

(b) $P(X \geq 2) = \frac{5}{20} + \frac{4}{20} + \frac{3}{20} + \frac{2}{20}$

Answer: (a) k = $\frac{1}{20}$ (b) $\frac{7}{10}$

10 Tangent condition for a line and a curve (AQA, Edexcel, OCR, MEI, Eduqas, CIE)

The line $y = -3x + k$ is a tangent to the curve

$y = x^2 + x$.

Find the value of k.

$x^2 + x = -3x + k$

$x^2 + 4x + (0 - k) = 0$

A tangent touches once: discriminant = 0

$(4)^2 - 4(0 - k) = 0$

Answer: k = −4