Day 294 **A Level Maths: C Tier 10-a-day: Model Solutions**

1 Constant velocity vector: when does it reach a point? (AQA, Edexcel, OCR, MEI, Eduqas)

A particle moves with constant velocity $(-2\mathbf{i} + 3\mathbf{j})$ m s^{-1}. It is at the point with position vector $(-\mathbf{i} + \mathbf{j})$ m at t = 0. Find the time when its position vector is $(-13\mathbf{i} + 19\mathbf{j})$ m.

$\mathbf{r} = \mathbf{r}_0 + \mathbf{v}t$

$(-13\mathbf{i} + 19\mathbf{j}) = (-\mathbf{i} + \mathbf{j}) + t(-2\mathbf{i} + 3\mathbf{j})$

Equating i components: $-13 = -1 + -2t$

Answer: 6 s

2 Position of a point relative to a circle (AQA, Edexcel, OCR, MEI, Eduqas, CIE)

A circle has equation $(x - 2)^2 + (y + 2)^2 = 16$.
Determine whether the point P(4, −7) is inside, outside or on the circle.

Distance^2 from the centre (2, −2) to P:

$(4 - 2)^2 + (-7 - (-2))^2 = 4 + 25 = 29$

$r^2 = 16$, and $29 > 16$.

Answer: P is outside the circle

3 F = ma: find the acceleration (All boards)

A horizontal force of 14 N acts on a box of mass 50 kg on a smooth horizontal surface.
Find the acceleration.

$F = ma$

$14 = 50\,a$

Answer: 0.28 m s^{-2}

4 Exponential graph through two points (AQA, Edexcel, OCR, MEI, Eduqas, CIE)

The diagram shows the curve $y = ab^x$, where $b > 0$.
Find the values of a and b.

Through (0, 2): $a \times b^0 = 2$, so $a = 2$.

Through (2, 18): $2b^2 = 18$

$b^2 = 9$, so $b = 3$ ($b > 0$)

Answer: a = 2, b = 3

5 Find the angle of a sector (All boards)

A sector has radius 4 cm and arc length 7.00 cm.
Find the angle of the sector in radians and the area of the sector.

$\theta = \frac{\text{arc length}}{r} = \frac{7.00}{4} = 1.75$

Area $= \frac{1}{2}r^2\theta = \frac{1}{2} \times 4^2 \times 1.75 = 14.0$ cm^2

Answer: θ = 1.75 radians; area 14.0 cm^2

6 Domain and range (AQA, Edexcel, OCR, MEI, Eduqas, CIE)

$f(x) = (x + 2)^2 - 4$, $x \geq 0$.
Find the range of f.

For $x \geq 0$ the least value is at $x = 0$ (the vertex $x = -2$ is not allowed).

$f(0) = 4 - 4 = 0$, and f increases for $x > 0$.

Answer: f(x) ≥ 0

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7 Integrate using partial fractions (All boards)

$\frac{-x+1}{(x+1)(x+3)} \equiv \frac{A}{x+1} + \frac{B}{x+3}$.

Find A and B, and hence find $\int \frac{-x+1}{(x+1)(x+3)}\,dx$.

$-x + 1 \equiv A(x + 3) + B(x + 1)$
$x = -1$ gives $A = 1$; $x = -3$ gives $B = -2$
$\int \left(\frac{1}{x+1} + \frac{-2}{x+3}\right) dx = 1 \ln|x + 1| + -2 \ln|x + 3| + c$
Answer: A = 1, B = −2; $\ln|x + 1| - 2 \ln|x + 3| + c$

8 Hypothesis test for a normal mean (AQA, Edexcel, OCR, MEI, Eduqas, CIE)

A population is $N(\mu, 2^2)$. H_0: $\mu = 165$, H_1: $\mu \neq 165$.

A sample of 36 has mean $\bar{x} = 165.4$.
Test at the 5% level. Critical value: $z = \pm 1.96$.

$z = (\bar{x} - \mu) \div (\sigma \div \sqrt{n}) = (165.4 - 165) \div (2 \div 6)$
$z = 1.20$
1.20 is outside the critical region, so do not reject H_0
Answer: z = 1.20; do not reject H_0

9 Quotient rule (All boards)

Use the quotient rule to find the derivative of

$y = \frac{x}{x+4}$.

Quotient rule: if $y = \frac{u}{v}$ then $dy/dx = \frac{u'v - uv'}{v^2}$

Differentiate the numerator and denominator, then substitute and simplify.

Answer: $dy/dx = \frac{4}{(x+4)^2}$

10 Two-tailed binomial test (AQA, Edexcel, OCR, MEI, Eduqas, CIE)

$X \sim B(10, p)$. H_0: $p = 0.4$, H_1: $p \neq 0.4$. In a sample, $x = 6$.
Test at the 5% significance level.

Two-tailed test, so compare with $0.05 \div 2 = 0.025$
$P(X \geq 6) = 0.166$
$0.166 > 0.025$, so do not reject H_0
Answer: Do not reject H_0 ($P(X \geq 6) = 0.166$)