Day 293 **A Level Maths: C Tier 10-a-day: Model Solutions**

1 Speed and direction from a velocity vector (AQA, Edexcel, OCR, MEI, Eduqas)

The velocity of a ball is $(-4\mathbf{i} + 3\mathbf{j})$ m s^{-1}, where $\mathbf{i}$ and $\mathbf{j}$ are perpendicular unit vectors.
(a) Find the speed.
(b) Find the acute angle between the direction of motion and the line of $\mathbf{i}$. Give 1 d.p.

(a) speed $= \sqrt{4^2 + 3^2} = \sqrt{25}$

(b) $\tan\theta = \frac{3}{4}$

$\theta = \tan^{-1}(\frac{3}{4})$

Answer: (a) 5 m s^{-1} (b) 36.9°

2 Vector equation of a line (3D) (CIE)

A line has equation $\mathbf{r} = \begin{pmatrix} -2 \\ 4 \\ 1 \end{pmatrix} + t\begin{pmatrix} 3 \\ 1 \\ -1 \end{pmatrix}$.
Show that the point $(-8, 2, 3)$ lies on the line.

Equate the first coordinates: $-2 + 3t = -8$, so $t = -2$
Check the others: $4 + 1 \times (-2) = 2$ and $1 + (-1) \times (-2) = 3$
All three agree, so the point is on the line
Answer: Shown ($t = -2$)

3 Arithmetic series: sum of terms (AQA, Edexcel, OCR, MEI, Eduqas, CIE)

Find the sum of the first 10 terms of the series
10 + 11 + 12 + 13 + ...

$a = 10$, $d = 1$, $n = 10$

$S_n = \frac{n}{2}[2a + (n - 1)d] = \frac{10}{2}[2(10) + 9 \times 1]$

$= 145$

Answer: 145

4 Chain rule: gradient at a point (All boards)

Find the gradient of the curve $y = \ln(2x + 2)$
at the point where $x = 2$.

Use the chain rule: $dy/dx = \frac{2}{2x + 2}$

At $x = 2$: $dy/dx = \frac{1}{3}$

Answer: $\frac{1}{3}$

5 Combined mean (AQA, Edexcel, OCR, MEI, Eduqas, CIE)

The mean of 40 times (seconds) of the Class A is 43. The mean of 50 times (seconds) of the Class B is 52.
Find the mean of all 90 values.

Total of first group = $40 \times 43 = 1720$
Total of second group = $50 \times 52 = 2600$
Mean = $4320 \div 90$
Answer: 48

6 Rewrite a sin x + b cos x in R form (All boards)

Express $3\sin x + 4\cos x$ in the form $R\cos(x - \alpha)$, where $R > 0$ and $0° < \alpha < 90°$.
Give α to 1 decimal place.

$R\cos(x - \alpha) = R\cos x\cos\alpha + R\sin x\sin\alpha$
$R\cos\alpha = 4$ and $R\sin\alpha = 3$

$R = \sqrt{3^2 + 4^2} = 5$, $\tan\alpha = \frac{3}{4}$, $\alpha = 36.9°$

Answer: $5\cos(x - 36.9°)$

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7 Reciprocal graphs and asymptotes (AQA, Edexcel, OCR, MEI, Eduqas, CIE)

Sketch $y = \frac{6}{x+4} + 1$.

Write down the equations of the asymptotes and the coordinates where the curve meets the axes.

Asymptotes: $x = -4$ and $y = 1$.

$y = 0$: $\frac{6}{x+4} = -1$, so $x = -10$

$x = 0$: $y = \frac{6}{4} + 1 = \frac{5}{2}$

Answer: $x = -4$, $y = 1$; $(-10, 0)$, $(0, \frac{5}{2})$

8 Sampling: advantages (AQA, Edexcel, OCR, MEI, Eduqas)

A researcher wants to find out the views of the students at a college.
Give one advantage and one disadvantage of using systematic sampling.

Advantage: it is quick and easy and spreads the sample evenly through the list.
Disadvantage: a repeating pattern in the list could cause bias, and a list is needed.
Answer: Advantage: quick and evenly spread. Disadvantage: a pattern in the list may bias

9 Differentiating displacement to find velocity (AQA, Edexcel, OCR, MEI, Eduqas, CIE)

A particle moves in a straight line. Its displacement from O at time t seconds is s metres, where $s = t^3 - 3t^2 + 6t$.
Find its velocity when $t = 3$.

$v = \frac{ds}{dt} = 3t^2 - 6t + 6$

When $t = 3$: $v = 3 \times 3^2 + -6 \times 3 + 6$

Answer: 15 m s^{-1}

10 Partial fractions (Edexcel, OCR, MEI, CIE, AQA, Eduqas)

Express $\frac{4x + 26}{(x + 5)(x + 2)}$ in partial fractions.

Write $\frac{4x + 26}{(x + 5)(x + 2)} = \frac{A}{(x + 5)} + \frac{B}{(x + 2)}$

$4x + 26 = A(x + 2) + B(x + 5)$

$x = -5$: $6 = A(-3)$ so $A = -2$

$x = -2$: $18 = B(3)$ so $B = 6$

Answer: $\frac{-2}{(x + 5)} + \frac{6}{(x + 2)}$