Day 292 **A Level Maths: C Tier 10-a-day: Model Solutions**

1 Free fall: dropped from a height (AQA, Edexcel, OCR, MEI, Eduqas, CIE)

A stone is dropped from rest from the top of a building of height 19.6 m. Take g = 9.8 m s^{-2} and ignore air resistance. Find (a) the time taken to reach the ground, (b) its speed on hitting the ground.

(a) $s = ut + \frac{1}{2}at^2$: $19.6 = \frac{1}{2} \times 9.8 \times t^2$

$19.6 = 4.9t^2$, so $t^2 = 4$

(b) $v = u + at = 9.8 \times 2$

Answer: (a) 2 s (b) 19.6 m s^{-1}

2 Integrate standard functions (All boards)

Find

(a) $\int 6 \sin 3x \, dx$

(b) $\int 6 \sec^2 3x \, dx$

Use the standard integrals and divide by the coefficient of x where needed.

(a) $= -2 \cos 3x + c$

(b) $= 2 \tan 3x + c$

Answer: (a) $-2 \cos 3x + c$ (b) $2 \tan 3x + c$

3 Hypotheses for a binomial test (AQA, Edexcel, OCR, MEI, Eduqas, CIE)

A company says that the probability a customer buys a ticket is 0.25.

A researcher thinks that the probability is different. Let p be the probability that customer buys a ticket.

Write down suitable hypotheses for a test on p.

H_0 states the claimed value; H_1 matches the researcher's belief.

Two-tailed test

Answer: H_0: $p = 0.25$, H_1: $p \neq 0.25$

4 Solving quadratics: surd answers (AQA, Edexcel, OCR, MEI, Eduqas, CIE)

Solve $x^2 + 6x - 6 = 0$, giving your answers in the form $p \pm \sqrt{q}$.

Complete the square: $(x + 3)^2 - 9 - 6 = 0$

$(x + 3)^2 = 15$

$x + 3 = \pm\sqrt{15}$

Answer: $x = -3 \pm \sqrt{15}$

5 Differentiating displacement to find velocity (AQA, Edexcel, OCR, MEI, Eduqas, CIE)

A body moves in a straight line. Its displacement from O at time t seconds is s metres, where $s = 3t^3 + 5t^2 + 6t$.

Find its velocity when t = 4.

$v = \frac{ds}{dt} = 9t^2 + 10t + 6$

When t = 4: $v = 9 \times 4^2 + 10 \times 4 + 6$

Answer: 190 m s^{-1}

6 Finding an unknown using the factor theorem (AQA, Edexcel, OCR, MEI, Eduqas, CIE)

$(x - 1)$ is a factor of $x^3 + kx^2 + 3x - 3$.

Find the value of k.

Let $f(x) = x^3 + kx^2 + 3x - 3$. Then $f(1) = 0$:

$1 + k + 0 = 0$

$k = -1$

Answer: $k = -1$

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7 Poisson distribution (CIE)

Cars pass a checkpoint at an average rate of 0.5 per hour, at random.
Find the probability of at most 3 in 3 hours, to 3 s.f.

Mean for 3 hours = 3 × 0.5 = 1.5
P(X ≤ 3) = P(0) + P(1) + ... + P(3), by calculator
Answer: 0.934

8 Find a and b in y = ab^x (All boards)

The curve $y = ab^x$ passes through (0, 5) and (3, $\frac{5}{8}$).

Find the values of a and b.

x = 0: $y = a \times b^0 = a$, so a = 5

x = 3: $5 \times b^3 = \frac{5}{8}$, so $b^3 = \frac{1}{8}$

$b = \frac{1}{2}$

Answer: a = 5, b = $\frac{1}{2}$

9 Gradient of a tangent to a circle (AQA, Edexcel, OCR, MEI, Eduqas, CIE)

The circle $(x + 1)^2 + (y + 2)^2 = 13$ passes through P(−3, −5).
Find the gradient of the tangent to the circle at P.

Centre C(−1, −2).

Gradient of CP = $\frac{-5 - (-2)}{-3 - (-1)} = \frac{3}{2}$

The tangent is perpendicular to CP: gradient = $-1 \div \frac{3}{2}$

Answer: $-\frac{2}{3}$

10 Trapezium rule from a table (AQA, Edexcel, OCR, MEI, Eduqas)

The table shows values of y = f(x).
x: 0 0.25 0.5 0.75 1 1.25
y: 8.4 6.6 3.7 4.3 5.5 5
Use the trapezium rule to estimate the area under the curve between x = 0 and x = 1.25.

h = 0.25 with 5 strips

Area ≈ $\frac{0.25}{2}[8.4 + 5 + 2(6.6 + 3.7 + 4.3 + 5.5)]$

= $\frac{0.25}{2} \times 53.6$

= 6.7

Answer: 6.7