Day 291 **A Level Maths: C Tier 10-a-day: Model Solutions**

1 Simplify using identities (All boards)

Simplify $\frac{1 - \cos^2 x}{\sin x}$.

$1 - \cos^2 x = \sin^2 x$, so the fraction is $\frac{\sin^2 x}{\sin x}$

Answer: sin x

2 Intercepts of a cubic graph (AQA, Edexcel, OCR, MEI, Eduqas, CIE)

Sketch $y = (x + 5)(x + 1)(x - 2)$. Write down the coordinates of the points where the curve meets the axes.

$y = 0$: $x = -5, -1, 2$
$x = 0$: $y = 5 \times 1 \times (-2) = -10$
Answer: $(-5, 0)$, $(-1, 0)$, $(2, 0)$, $(0, -10)$

3 Catching up: constant speed against acceleration (AQA, Edexcel, OCR, MEI, Eduqas, CIE)

Car P passes a point O at constant speed 10 m s^{-1}.

At that instant car Q starts from rest at O with constant acceleration 2 m s^{-2} in the same direction. Find the time when Q catches up P.

Distances equal: $10\,t = \frac{1}{2} \times 2 \times t^2$

$t(10 - 1\,t) = 0$, so $t = 0$ or $t = 10 \div 1$
Answer: 10 s

4 Standard deviation from raw data (AQA, Edexcel, OCR, MEI, Eduqas, CIE)

Here are 7 values: 10, 16, 20, 10, 3, 14, 9

Find the mean and the standard deviation, to 3 s.f.

$\Sigma x = 82$, $\Sigma x^2 = 1142$, $n = 7$
Mean $= 82 \div 7 = 11.71$

Variance $= \frac{1142}{7} - (11.71)^2 = 25.92$

Standard deviation $= \sqrt{25.92}$
Answer: mean = 11.7, standard deviation = 5.09

5 Resolving a force into components (All boards)

A rope pulls a box with a force of 40 N at an angle of 30° above the horizontal. Find the horizontal and vertical components of the force, to 3 s.f.

Horizontal $= 40 \cos 30°$
Vertical $= 40 \sin 30°$
Answer: horizontal 34.6 N, vertical 20.0 N

6 Change of sign to locate a root (All boards)

$f(x) = e^x - 1x - 3$.

Show that $f(x) = 0$ has a root between $x = 1$ and $x = 2$.

$f(1) = -1.28$
$f(2) = 2.39$
f is continuous and there is a change of sign, so a root lies in the interval
Answer: f(1) = −1.28, f(2) = 2.39: change of sign

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7 Binomial expansion: non-integer powers (Edexcel, AQA, OCR, MEI, Eduqas, CIE)

Expand $(1 - 3x)^{\frac{1}{3}}$ in ascending powers of x up to and including the term in x^2. State the range of values of x for which the expansion is valid.

$(1 + y)^n = 1 + ny + \frac{n(n-1)}{2!}y^2 + \ldots$ with $n = \frac{1}{3}$, $y = -3x$

$= 1 + \frac{1}{3}(-3x) + \frac{\frac{1}{3}((-\frac{2}{3}))}{2}(-3x)^2$

$= 1 - x - x^2$

Valid when $|-3x| < 1$, i.e. $|x| < \frac{1}{3}$

Answer: $1 - x - x^2$; $|x| < \frac{1}{3}$

8 Outliers (Edexcel, OCR, MEI)

For a data set $Q_1 = 13$ and $Q_3 = 37$.

A value is an outlier if it is more than 1.5 × IQR above Q_3 or below Q_1.

Determine whether 34 and −27 are outliers.

IQR = 37 − 13 = 24, so 1.5 × IQR = 36

Lower limit = 13 − 36 = −23; upper limit = 37 + 36 = 73

34 is not an outlier; −27 is an outlier

Answer: 34 is not an outlier; −27 is an outlier

9 Arithmetic series: sum of terms (AQA, Edexcel, OCR, MEI, Eduqas, CIE)

Find the sum of the first 16 terms of the series

3 + 9 + 15 + 21 + ...

a = 3, d = 6, n = 16

$S_n = \frac{n}{2}[2a + (n-1)d] = \frac{16}{2}[2(3) + 15 \times 6]$

= 768

Answer: 768

10 Find the angle of a sector (All boards)

A sector has radius 10 cm and area 25.0 cm^2.

Find the angle of the sector in radians and the arc length.

$\frac{1}{2} \times 10^2 \times \theta = 25.0$, so $\theta = 0.5$

Arc length = $r\theta = 10 \times 0.5 = 5.00$ cm

Answer: θ = 0.5 radians; arc length 5.00 cm