**Day 290** **A Level Maths: C Tier 10-a-day: Model Solutions**

## 1 Scatter diagrams and correlation (AQA, Edexcel, OCR, MEI, Eduqas, CIE)

The scatter diagram shows engine size and fuel used for a group.
Describe the correlation.

The points lie close to a line with positive gradient.
As engine size increases, fuel used tends to increase.
**Answer: strong positive correlation**

## 2 Geometric series: sum to infinity (AQA, Edexcel, OCR, MEI, Eduqas, CIE)

A geometric series has first term 11 and common ratio $-\frac{1}{2}$.

Find the sum to infinity.

$|r| < 1$, so the series converges.

$S_\infty = \frac{a}{1-r} = \frac{11}{1-(-\frac{1}{2})}$

$= \frac{22}{3}$

**Answer: $\frac{22}{3}$**

## 3 Trapezium rule: strips shown (diagram) (AQA, Edexcel, OCR, MEI, Eduqas)

The diagram shows the curve $y = e^x$ from $x = 0$ to $x = 3$ divided into 6 strips of equal width.
Use the trapezium rule with these strips to estimate the shaded area to 3 s.f.

$h = 0.5$; y values: 1.000, 1.649, 2.718, 4.482, 7.389, 12.18, 20.09

Area $\approx \frac{0.5}{2}[1.000 + 20.09 + 2(28.42)]$

$= 19.5$

**Answer: 19.5**

## 4 Inverse functions (AQA, Edexcel, OCR, MEI, Eduqas, CIE)

$f(x) = \frac{1}{x+2}$, $x \neq -2$.

Find $f^{-1}(x)$.

$y = \frac{1}{x+2}$, so $y(x+2) = 1$

$x + 2 = \frac{1}{y}$, so $x = \frac{1}{y} - 2$

**Answer: $f^{-1}(x) = \frac{1}{x} - 2$**

## 5 Two-stage motion: acceleration then constant speed (AQA, Edexcel, OCR, MEI, Eduqas, CIE)

A train starts from rest and accelerates uniformly at 1 m $s^{-2}$ for 8 s, then travels at constant speed for a further 20 s. Find the total distance travelled.

Speed reached: $v = 1 \times 8 = 8$ m $s^{-1}$

Stage 1: $s = \frac{1}{2} \times 8 \times 8 = 32$

Stage 2: $s = 8 \times 20 = 160$

**Answer: 192 m**

## 6 Solve a logarithmic equation (two logs) (All boards)

Solve $\ln(x + 9) - \ln x = \ln 4$.

$\ln \frac{x+9}{x} = \ln 4$, so $\frac{x+9}{x} = 4$

$x + 9 = 4x$, so $9 = 3x$

$x = 3$

**Answer: $x = 3$**

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## 7 Moments: reactions on a uniform beam (AQA, Edexcel, OCR, MEI)

A uniform beam AB of mass 15 kg and length 6 m rests horizontally on supports at A and B. A load of mass 25 kg is placed on the beam 4.5 m from A. Take g = 9.8 m $s^{-2}$. Find the reaction at B.

Moments about A:

$R_B \times 6 = 15 \times 9.8 \times 3 + 25 \times 9.8 \times 4.5$

$R_B \times 6 = 1543.5$

**Answer: 257.25 N**

## 8 Finding a quadratic from its turning point (AQA, Edexcel, OCR, MEI, Eduqas, CIE)

The diagram shows a sketch of a quadratic curve.

Find its equation in the form $y = a(x - p)^2 + q$.

The turning point is $(-1, 0)$, so $y = a(x + 1)^2 + 0$.

The curve passes through $(0, -2)$:

$-2 = a(1)^2 + 0$, so $-2 = 1\ a$

$a = -2$

**Answer: $y = -2(x + 1)^2 + 0$**

## 9 Binomial test using a given probability (AQA, Edexcel, OCR, MEI, Eduqas, CIE)

$X \sim B(20, p)$. $H_0$: $p = 0.3$, $H_1$: $p < 0.3$.

In a sample, $x = 2$. It is given that $P(X \leq 2) = 0.0355$.

Test $H_0$ at the 5% significance level.

Compare 0.0355 with 0.05

0.0355 < 0.05, so reject $H_0$

**Answer: Reject $H_0$: evidence that p is less than 0.3**

## 10 Newton-Raphson: one or two steps (AQA, Edexcel, OCR, MEI, Eduqas)

$f(x) = x^2 - 2$.

Use the Newton-Raphson method with $x_0 = 2$ to find $x_1$ and $x_2$,

giving your answer to 3 decimal places.

$f'(x) = 2x$

$$x_{n+1} = x_n - \frac{f(x_n)}{f'(x_n)}$$

$$x_1 = 2 - \frac{2.000}{4.000} = 1.5000$$

$$x_2 = 1.5000 - \frac{0.2500}{3.000} = 1.4167$$

**Answer: $x_1 = 1.500$, $x_2 = 1.417$**