Day 29 **A Level Maths: C Tier 10-a-day: Model Solutions**

1 Iteration $x_{n+1} = g(x_n)$ (All boards)

$x_{n+1} = (3x_n + 7)^{\frac{1}{3}}$, with $x_0 = 1$.
Find x_1, x_2 and x_3 to 3 decimal places.

$x_1 = 2.1544$
$x_2 = 2.3789$
$x_3 = 2.4180$
Answer: $x_1 = 2.154$, $x_2 = 2.379$, $x_3 = 2.418$

2 Braking: deceleration and time (AQA, Edexcel, OCR, MEI, Eduqas, CIE)

A cyclist travelling at 24 m s^{-1} brakes uniformly and comes to rest after travelling 72 m.
(a) Find the deceleration.
(b) Find the time taken to stop.

(a) $v^2 = u^2 + 2as$: $0 = 24^2 + 2 \times a \times 72$
so $a = -4$, a deceleration of 4
(b) $s = \frac{(u + v)}{2}$ t: $72 = 12$ t
Answer: (a) 4 m s^{-2} (b) 6 s

3 Counting: arrangements and selections (CIE)

How many different arrangements are there of the letters of the word BALLOON?

7 letters, with repeats: L appears 2 times, O appears 2 times
7! ÷ (2!×2!)
Answer: 1260

4 Position of a point relative to a circle (AQA, Edexcel, OCR, MEI, Eduqas, CIE)

A circle has equation $(x - 3)^2 + (y - 3)^2 = 36$.
Determine whether the point P(2, −2) is inside, outside or on the circle.

Distance^2 from the centre (3, 3) to P:
$(2 - 3)^2 + (-2 - 3)^2 = 1 + 25 = 26$
$r^2 = 36$, and $26 < 36$.
Answer: P is inside the circle

5 Degrees and radians (All boards)

(a) Convert 30° to radians, giving your answer in terms of π.
(b) Convert $\frac{5\pi}{12}$ radians to degrees.

(a) $30° = 30 \times \frac{\pi}{180} = \frac{\pi}{6}$
(b) $\frac{5\pi}{12}$ radians $= \frac{5\pi}{12} \times \frac{180}{\pi} = 75°$
Answer: (a) $\frac{\pi}{6}$ radians (b) 75°

6 Algebraic division (AQA, Edexcel, OCR, MEI, Eduqas, CIE)

Divide $f(x) = x^3 - 17x - 5$ by $(x + 4)$.
Find the quotient and the remainder.

Use synthetic division with $a = -4$:
Bottom row: 1, −4, −1, −1
Quotient: $x^2 - 4x - 1$
Remainder: −1
Answer: Quotient $x^2 - 4x - 1$, remainder −1

7 Solve trig equation in radians (exact) (All boards)

Solve $\tan x = -1$ for $0 \leq x \leq 2\pi$.
Give your answers in terms of π.

Reference angle: $\frac{\pi}{4}$

tan is negative in the second and fourth quadrants.

$x = \frac{3\pi}{4}, \frac{7\pi}{4}$

Answer: $x = \frac{3\pi}{4}, \frac{7\pi}{4}$

8 Straight line through two points (AQA, Edexcel, OCR, MEI, Eduqas, CIE)

Find an equation of the line through $A(-2, -1)$ and $B(-6, 2)$, in the form $ax + by + c = 0$, where a, b and c are integers.

Gradient $= \frac{2 - (-1)}{-6 - (-2)} = -\frac{3}{4}$

$y - (-1) = -\frac{3}{4}(x - (-2))$

Multiply out and rearrange to integer coefficients.

Answer: $3x + 4y + 10 = 0$

9 Starting from rest: find the time (AQA, Edexcel, OCR, MEI, Eduqas, CIE)

A train starts from rest and moves in a straight line with constant acceleration 1.5 m s^{-2}.
It travels 48 m. Find the time taken.

$s = ut + \frac{1}{2}at^2$ with $u = 0$

$48 = \frac{1}{2} \times 1.5 \times t^2$

$t^2 = 64$

Answer: 8 s

10 Normal approximation to the binomial (CIE)

$X \sim B(100, 0.25)$. Use a normal approximation to find $P(X \leq 25)$, to 3 s.f.

$np = 25$ and $npq = 18.75$, so use $Y \sim N(25, 18.75)$
Continuity correction: $P(Y < 25.5)$

$z = (25.5 - 25) \div \sqrt{18.75}$

Answer: 0.546