Day 289 **A Level Maths: C Tier 10-a-day: Model Solutions**

1 Independent events (AQA, Edexcel, OCR, MEI, Eduqas, CIE)

P(A) = 0.8, P(B) = 0.8 and P(A ∩ B) = 0.64.
(a) Find P(A | B).
(b) Are A and B independent? Give a reason.

(a) P(A | B) = P(A ∩ B) ÷ P(B) = 0.64 ÷ 0.8 = 0.8
(b) P(A) × P(B) = 0.8 × 0.8 = 0.64
This equals P(A ∩ B) = 0.64, so A and B are independent
Answer: (a) 0.8 (b) independent, as P(A) × P(B) = P(A ∩ B)

2 Solve basic trig equation (exact, degrees) (All boards)

Solve $\sin x = \frac{\sqrt{2}}{2}$ for $0° \leq x \leq 360°$.

Reference angle: $\sin 45° = \frac{\sqrt{2}}{2}$
sin is positive in the first and second quadrants.
x = 45°, 135°
Answer: x = 45°, 135°

3 Composite functions (AQA, Edexcel, OCR, MEI, Eduqas, CIE)

f(x) = 2x + 6 and $g(x) = x^2 - 1$.
Find (a) fg(x), (b) gf(x).

$fg(x) = f(x^2 - 1) = 2x^2 + 4$
$gf(x) = (2x + 6)^2 - 1 = 4x^2 + 24x + 35$
Answer: (a) $2x^2 + 4$ (b) $4x^2 + 24x + 35$

4 Constant acceleration: time to reach a speed (AQA, Edexcel, OCR, MEI, Eduqas, CIE)

A cyclist moving in a straight line increases its speed from 1 m s^{-1} to 11.5 m s^{-1} with constant acceleration 1.5 m s^{-2}.
Find the time taken.

v = u + at
11.5 = 1 + 1.5 t
Answer: 7 s

5 Exponential graphs (AQA, Edexcel, OCR, MEI, Eduqas, CIE)

Sketch $y = 5^x - 25$.
Write down the equation of the asymptote and the coordinates where the curve meets the axes.

As $x \to -\infty$, $5^x \to 0$, so $y \to -25$: asymptote $y = -25$.
x = 0: $y = 5^0 - 25 = -24$
y = 0: $5^x = 25$, so $5^x = 25$ and x = 2
Answer: $y = -25$; (0, −24), (2, 0)

6 Parametric differentiation (AQA, Edexcel, OCR, MEI, Eduqas, CIE)

$x = 4t^2 + 3$, $y = 3t^3 - t$.
Find dy/dx in terms of t, and its value when t = 1.

dx/dt = 8t, $dy/dt = 9t^2 - 1$

$dy/dx = \frac{dy}{dt \div dx/dt} = \frac{9t^2 - 1}{8t}$
At t = 1: dy/dx = 1
Answer: $dy/dx = \frac{9t^2 - 1}{8t}$; 1

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7 Constant acceleration: $v^2 = u^2 + 2as$ (AQA, Edexcel, OCR, MEI, Eduqas, CIE)

A particle moves in a straight line with constant acceleration. Its speed increases from 1 m s^{-1} to 5 m s^{-1} as it travels 24 m. Find the acceleration.

$v^2 = u^2 + 2as$

$5^2 = 1^2 + 2 \times a \times 24$

$24 = 48\ a$

Answer: 0.5 m s^{-2}

8 Natural logs and e^x (All boards)

Solve 3 ln x = 4.

Give your answer to 3 significant figures.

$\ln x = \frac{4}{3}$

$x = e^{\frac{4}{3}} = 3.79$

Answer: x = 3.79

9 Selection without replacement (AQA, Edexcel, OCR, MEI, Eduqas, CIE)

A bag has 5 red and 2 blue sweets. Two are taken at random without replacement.

Find the probability that at least one of them is red.

$P(\text{no red}) = P(BB) = \frac{2}{7} \times \frac{1}{6} = \frac{1}{21}$

$P(\text{at least one red}) = 1 - \frac{1}{21}$

Answer: $\frac{20}{21}$

10 Tangent condition for a line and a curve (AQA, Edexcel, OCR, MEI, Eduqas, CIE)

The line $y = 3x + k$ is a tangent to the curve

$y = x^2 + x - 1$.

Find the value of k.

$x^2 + x - 1 = 3x + k$

$x^2 - 2x + (-1 - k) = 0$

A tangent touches once: discriminant = 0

$(-2)^2 - 4(-1 - k) = 0$

Answer: k = −2