Day 288 **A Level Maths: C Tier 10-a-day: Model Solutions**

1 Natural logs and e^x (All boards)

Solve (a) $e^x = 12$ (b) $\ln x = 0.5$
Give answers to 3 significant figures.

(a) $x = \ln 12 = 2.48$

(b) $x = e^{0.5} = 1.65$

Answer: (a) x = 2.48 (b) x = 1.65

2 Parametric equations: finding points (Edexcel, OCR, MEI, CIE, AQA, Eduqas)

A curve has parametric equations
$x = 4t - 5$, $y = 2t^2 - 6$.
The point P has y-coordinate -4 and positive t.
Find the coordinates of P.

$2t^2 - 6 = -4$, so $t^2 = 1$, giving $t = 1$ ($t > 0$)
$x = 4 \times 1 - 5 = -1$
Answer: (−1, −4)

3 Large data set: handling readings (AQA, Edexcel, OCR, MEI, Eduqas)

Seven daily rainfall readings (mm) from the Large Data Set are: 2, tr, 3.2, 2.8, tr, 3.6, 3.4.
Here 'tr' means a trace, which is treated as 0.
Find the mean daily rainfall to 3 s.f.

Replace each 'tr' by 0
Total = 15
Mean = 15 ÷ 7
Answer: 2.14 mm

4 Constant velocity vector: when does it reach a point? (AQA, Edexcel, OCR, MEI, Eduqas)

A particle moves with constant velocity
$(-2\mathbf{i} + 3\mathbf{j})$ m s^{-1}. It is at the point with position vector $(\mathbf{i} + 2\mathbf{j})$ m at t = 0. Find the time when its position vector is $(-11\mathbf{i} + 20\mathbf{j})$ m.

$\mathbf{r} = \mathbf{r}_0 + \mathbf{v}t$
$(-11\mathbf{i} + 20\mathbf{j}) = (\mathbf{i} + 2\mathbf{j}) + t(-2\mathbf{i} + 3\mathbf{j})$
Equating i components: $-11 = 1 + -2t$
Answer: 6 s

5 Vector equation of a line (3D) (CIE)

A line has equation $\mathbf{r} = \begin{pmatrix}1\\3\\1\end{pmatrix} + t\begin{pmatrix}3\\3\\-2\end{pmatrix}$.
Show that the point (13, 15, −7) lies on the line.

Equate the first coordinates: $1 + 3t = 13$, so $t = 4$
Check the others: $3 + 3 \times 4 = 15$ and $1 + (-2) \times 4 = -7$
All three agree, so the point is on the line
Answer: Shown (t = 4)

6 Arithmetic sequences: nth term (AQA, Edexcel, OCR, MEI, Eduqas, CIE)

In an arithmetic sequence $u_5 = 11$ and $u_7 = 15$.
Find the first term and the common difference.

Subtract: $(7 - 5)d = 15 - 11$
$2d = 4$, so $d = 2$
$a = u_5 - 4d = 11 - 8 = 3$
Answer: a = 3, d = 2

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7 Partial fractions: find the constants (All boards)

Express $\frac{-x+1}{(x+2)(x+5)}$ in the form $\frac{A}{x+2}+\frac{B}{x+5}$.

Find the values of A and B.

$-x+1 \equiv A(x+5)+B(x+2)$

$x=-2$: $3=A(3)$, so $A=1$

$x=-5$: $6=B(-3)$, so $B=-2$

Answer: $A=1$, $B=-2$

8 Normal approximation to the binomial (CIE)

$X \sim B(100, 0.5)$. Use a normal approximation to find $P(X \geq 55)$, to 3 s.f.

$np = 50$ and $npq = 25$, so use $Y \sim N(50, 25)$

Continuity correction: $P(Y > 54.5)$

$z = (54.5 - 50) \div \sqrt{25}$

Answer: 0.184

9 Particle held by a string and a horizontal force (All boards)

A lamp of weight 48 N hangs from a light string, pulled aside by a horizontal force P N so that the string makes an angle θ with the vertical, where $\tan\theta = \frac{5}{12}$. Find P and the tension T N.

Vertically: $T\cos\theta = 48$

Horizontally: $T\sin\theta = P$

Dividing: $P = 48\tan\theta$; $T = \sqrt{48^2 + P^2}$

Answer: $P=20$, $T=52$

10 Tangent condition for a line and a curve (AQA, Edexcel, OCR, MEI, Eduqas, CIE)

The line $y = 4x + k$ is a tangent to the curve

$y = x^2 + 8x - 3$.

Find the value of k.

$x^2 + 8x - 3 = 4x + k$

$x^2 + 4x + (-3 - k) = 0$

A tangent touches once: discriminant = 0

$(4)^2 - 4(-3 - k) = 0$

Answer: $k=-7$