Day 287 **A Level Maths: C Tier 10-a-day: Model Solutions**

1 Parametric equations: finding points (Edexcel, OCR, MEI, CIE, AQA, Eduqas)

A curve has parametric equations

$x = t + 3$, $y = 2t^2 - 6$.

Find the coordinates of the point where $t = 1$.

$x = 1 \times 1 + 3 = 4$

$y = 2 \times 1^2 - 6 = -4$

Answer: (4, −4)

2 Chain rule: gradient at a point (All boards)

Find the gradient of the curve $y = (x^2 + 4)^2$
at the point where $x = 0$.

Use the chain rule: $dy/dx = 2(x^2 + 4)^1 \times 2x$

At $x = 0$: $dy/dx = 0$

Answer: 0

3 Coding (AQA, Edexcel, OCR, MEI, Eduqas, CIE)

The values x are coded using $y = (x - 20) \div 5$.

The coded values have mean $\bar{y} = 0.6$ and standard deviation 0.5.
Find the mean and standard deviation of x.

$x = 20 + 5y$

Mean: $20 + 5 \times 0.6 = 23$

Standard deviation: $5 \times 0.5 = 2.5$ (adding 20 has no effect)

Answer: mean = 23, standard deviation = 2.5

4 Newton-Raphson: one or two steps (AQA, Edexcel, OCR, MEI, Eduqas)

$f(x) = e^x - 4$.

Use the Newton-Raphson method with $x_0 = 1$ to find x_1 and x_2,
giving your answer to 3 decimal places.

$f'(x) = e^x$

$$x_{n+1} = x_n - \frac{f(x_n)}{f'(x_n)}$$

$$x_1 = 1 - \frac{-1.282}{2.718} = 1.4715$$

$$x_2 = 1.4715 - \frac{0.3558}{4.356} = 1.3898$$

Answer: $x_1 = 1.472$, $x_2 = 1.390$

5 Equation of a circle from a diagram (AQA, Edexcel, OCR, MEI, Eduqas, CIE)

The diagram shows a circle with the centre marked.
The circle touches the x-axis. Find the equation of the circle.

The circle touches the x-axis, so the radius equals the
y-coordinate of the centre: $r = 3$.

Centre $(-4, 3)$: $(x - a)^2 + (y - b)^2 = r^2$

Answer: $(x + 4)^2 + (y - 3)^2 = 9$

6 Catching up: constant speed against acceleration (AQA, Edexcel, OCR, MEI, Eduqas, CIE)

Car A passes a point O at constant speed 5.5 m s^{-1}.

At that instant car B starts from rest at O with constant acceleration 1 m s^{-2} in the same direction. Find the time when B catches up A.

Distances equal: $5.5\,t = \frac{1}{2} \times 1 \times t^2$

$t(5.5 - 0.5\,t) = 0$, so $t = 0$ or $t = 5.5 \div 0.5$

Answer: 11 s

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7 Normal approximation to the binomial (CIE)

X ~ B(60, 0.3). Use a normal approximation to find P(X ≤ 18), to 3 s.f.

np = 18 and npq = 12.6, so use Y ~ N(18, 12.6)
Continuity correction: P(Y < 18.5)
$z = (18.5 - 18) \div \sqrt{12.6}$
Answer: 0.556

8 Stone thrown down from a cliff (AQA, Edexcel, OCR, MEI, Eduqas, CIE)

A stone is thrown vertically downwards at 5 m s^{-1} from the top of a cliff and hits the sea after 4 s. Taking g = 9.8 m s^{-2}, find the height of the cliff.

$s = ut + \frac{1}{2}at^2$

$s = 5 \times 4 + \frac{1}{2} \times 9.8 \times 4^2$

$s = 20 + 78.4$
Answer: 98.4 m

9 Product rule (All boards)

Use the product rule to differentiate
$y = x^2 \sin 4x$.

$u = x^2$, $v = \sin 4x$
$u' = 2x^1$, $v' = 4 \cos 4x$
dy/dx = u′v + uv′, which simplifies to the answer below
Answer: $dy/dx = x^1(2 \sin 4x + 4x \cos 4x)$

10 Real roots and the discriminant (AQA, Edexcel, OCR, MEI, Eduqas, CIE)

Find the set of values of k for which
$x^2 + kx + 4 = 0$ has two distinct real roots.

$b^2 - 4ac > 0$
$k^2 - 16 > 0$, so $k^2 > 16$
Answer: $k < -4$ or $k > 4$