Day 286 **A Level Maths: C Tier 10-a-day: Model Solutions**

1 Speed and direction from a velocity vector (AQA, Edexcel, OCR, MEI, Eduqas)

The velocity of a particle is $(3\mathbf{i} + 4\mathbf{j})$ m s^{-1}, where $\mathbf{i}$ and $\mathbf{j}$ are perpendicular unit vectors.
(a) Find the speed.
(b) Find the acute angle between the direction of motion and the line of $\mathbf{i}$. Give 1 d.p.

(a) speed $= \sqrt{3^2 + 4^2} = \sqrt{25}$

(b) $\tan\theta = \frac{4}{3}$

$\theta = \tan^{-1}(\frac{4}{3})$

Answer: (a) 5 m s^{-1} (b) 53.1°

2 Factorised trig equation (All boards)

Solve $(2\cos x - 1)(2\cos x + 1) = 0$ for $0° \leq x \leq 360°$.

Either $2\cos x - 1 = 0$ or $2\cos x + 1 = 0$

$\cos x = \frac{1}{2}$: $x = 60°, 300°$

$\cos x = -\frac{1}{2}$: $x = 120°, 240°$

Answer: x = 60°, 120°, 240°, 300°

3 Arithmetic series: sum of terms (AQA, Edexcel, OCR, MEI, Eduqas, CIE)

An arithmetic series has first term −5, last term 142 and 22 terms. Find the sum of the series.

$S_n = \frac{n}{2}(a + l) = \frac{22}{2}(-5 + 142)$

$= 1507$

Answer: 1507

4 Discrete random variable: finding k (AQA, Edexcel, OCR, MEI, Eduqas, CIE)

The discrete random variable X has $P(X = x) = k(x^2 + 1)$ for $x = 1, 2, 3, 4$.
(a) Find the value of k.
(b) Find $P(X \geq 2)$.

(a) Probabilities sum to 1: $k(2 + 5 + 10 + 17) = 1$

$k = \frac{1}{34}$

(b) $P(X \geq 2) = \frac{5}{34} + \frac{10}{34} + \frac{17}{34}$

Answer: (a) k = $\frac{1}{34}$ (b) $\frac{16}{17}$

5 Projectile launched with velocity components (AQA, Edexcel, OCR, MEI)

A ball is projected from level ground with velocity $(7\mathbf{i} + 9.8\mathbf{j})$ m s^{-1} ($\mathbf{i}$ horizontal, $\mathbf{j}$ vertically up). Take g = 9.8 m s^{-2}.
(a) Find the time of flight.
(b) Find the greatest height reached.

(a) Vertically: $0 = 9.8t - 4.9t^2$

$t = \frac{9.8}{4.9} = 2$

(b) $0 = 9.8^2 - 2 \times 9.8 \times H$

$H = \frac{96.04}{19.6}$

Answer: (a) 2 s (b) 4.9 m

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6 Recurrence relations (AQA, Edexcel, OCR, MEI, Eduqas, CIE)

A sequence is defined by $u_{n+1} = \frac{1}{3} u_n + 4$.

The sequence converges to a limit L. Find L.

As $n \to \infty$, u_{n+1} and u_n both tend to L.

$L = \frac{1}{3} L + 4$

$L(1 - \frac{1}{3}) = 4$

Answer: L = 6

7 Find a and b in y = ab^x (All boards)

The curve $y = ab^x$ passes through (0, 3) and (4, 48).
Find the values of a and b.

$x = 0$: $y = a \times b^0 = a$, so $a = 3$

$x = 4$: $3 \times b^4 = 48$, so $b^4 = 16$

$b = 2$

Answer: a = 3, b = 2

8 Critical region (AQA, Edexcel, OCR, MEI, Eduqas, CIE)

$X \sim B(20, p)$. H_0: $p = 0.5$, H_1: $p < 0.5$.

Find the critical region for a test at the 10% significance level.

$P(X \leq 6) = 0.0577 \leq 0.1$

$P(X \leq 7) = 0.132 > 0.1$

Answer: X ≤ 6

9 Perpendicular bisector (AQA, Edexcel, OCR, MEI, Eduqas, CIE)

A is (5, −3) and B is (13, 3).
Find the equation of the perpendicular bisector of AB, in the form ax + by = c with integer a, b and c.

Midpoint of AB = (9, 0)

Gradient of AB = $\frac{6}{8} = \frac{3}{4}$, so perpendicular gradient = $-\frac{4}{3}$

$y - 0 = -\frac{4}{3}(x - 9)$

Multiply out and rearrange.

Answer: 4x + 3y = 36

10 Simplify using identities (All boards)

Simplify $\frac{\sin x \cos x}{\tan x}$.

$\tan x = \frac{\sin x}{\cos x}$, so this is $\sin x \cos x \times \frac{\cos x}{\sin x}$

Answer: $\cos^2 x$