Day 285 **A Level Maths: C Tier 10-a-day: Model Solutions**

1 Iteration $x_{n+1} = g(x_n)$ (All boards)

$x_{n+1} = \ln(x_n + 2) + 1$, with $x_0 = 2$.

Find x_1, x_2 and x_3 to 3 decimal places.

$x_1 = 2.3863$

$x_2 = 2.4785$

$x_3 = 2.4993$

Answer: $x_1 = 2.386$, $x_2 = 2.478$, $x_3 = 2.499$

2 Binomial: P(X = k) (AQA, Edexcel, OCR, MEI, Eduqas, CIE)

X ~ B(7, 0.2). Find P(X = 1), giving your answer to 3 s.f.

$P(X = 1) = {}^7C_1 \times 0.2^1 \times 0.8^6$

$= 7 \times 0.2^1 \times 0.8^6$

Answer: 0.367

3 Equilibrium of forces given as vectors (AQA, Edexcel, OCR, MEI, Eduqas)

Three forces $(-3\mathbf{i})$ N, $(4\mathbf{i} - 3\mathbf{j})$ N and $(p\mathbf{i} + q\mathbf{j})$ N act on a particle which is in equilibrium. $\mathbf{i}$ and $\mathbf{j}$ are perpendicular unit vectors. Find the values of p and q.

Sum of forces = 0:

i components: $p + (-3) + (4) = 0$

j components: $q + (0) + (-3) = 0$

Answer: $p = -1$, $q = 3$

4 Parametric differentiation (AQA, Edexcel, OCR, MEI, Eduqas, CIE)

$x = 2\cos t$, $y = 2\sin t$.

Find dy/dx in terms of t, and its value when $t = \frac{\pi}{4}$.

$dx/dt = -2\sin t$, $dy/dt = 2\cos t$

$dy/dx = \frac{2\cos t}{-2\sin t} = -\frac{2}{2}\cot t$

At $t = \frac{\pi}{4}$, $\cot t = 1$, so $dy/dx = -1$

Answer: $dy/dx = -\frac{2}{2}\cot t$; -1

5 Equation of a circle (AQA, Edexcel, OCR, MEI, Eduqas, CIE)

A is (−3, −1) and B is (5, 3). AB is a diameter of a circle.

Find the equation of the circle.

Centre = midpoint of AB = (1, 1)

$r^2 = (5 - 1)^2 + (3 - 1)^2 = 20$

Equation: $(x - a)^2 + (y - b)^2 = r^2$

Answer: $(x - 1)^2 + (y - 1)^2 = 20$

6 Addition formulae: exact values (All boards)

By writing 75° as 45° + 30°, find the exact value of sin 75°.

$\sin(45° + 30°) = \sin 45° \cos 30° + \cos 45° \sin 30°$

$= \frac{\sqrt{2}}{2} \times \frac{\sqrt{3}}{2} + \frac{\sqrt{2}}{2} \times \frac{1}{2}$

Simplify: $\frac{\sqrt{6} + \sqrt{2}}{4}$

Answer: $\frac{\sqrt{6} + \sqrt{2}}{4}$

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7 Velocity-time graph: acceleration and distance (AQA, Edexcel, OCR, MEI, Eduqas, CIE)

The graph shows the velocity of a lift over 32 seconds as it moves in a straight line.
(a) Find its acceleration in the first 8 s.
(b) Find the total distance travelled.

(a) acceleration = gradient = $\frac{8}{8}$

(b) distance = area of trapezium

= $\frac{1}{2}(20 + 32) \times 8$

Answer: (a) 1 m s^{-2} (b) 208 m

8 Binomial expansion: non-integer powers (Edexcel, AQA, OCR, MEI, Eduqas, CIE)

Expand $(1 + 2x)^{\frac{1}{2}}$ in ascending powers of x up to and including the term in x^2. State the range of values of x for which the expansion is valid.

$(1 + y)^n = 1 + ny + \frac{n(n-1)}{2!}y^2 + \ldots$ with $n = \frac{1}{2}$, $y = 2x$

$= 1 + \frac{1}{2}(2x) + \frac{\frac{1}{2}((-\frac{1}{2}))}{2}(2x)^2$

$= 1 + x - \frac{1}{2}x^2$

Valid when $|2x| < 1$, i.e. $|x| < \frac{1}{2}$

Answer: $1 + x - \frac{1}{2}x^2$; $|x| < \frac{1}{2}$

9 Normal approximation to the binomial (CIE)

X ~ B(50, 0.2). Use a normal approximation to find P(X ≤ 7), to 3 s.f.

np = 10 and npq = 8, so use Y ~ N(10, 8)
Continuity correction: P(Y < 7.5)
$z = (7.5 - 10) \div \sqrt{8}$
Answer: 0.188

10 Tangent condition for a line and a curve (AQA, Edexcel, OCR, MEI, Eduqas, CIE)

The line $y = -x + k$ is a tangent to the curve
$y = x^2 + x + 3$.
Find the value of k.

$x^2 + x + 3 = -x + k$
$x^2 + 2x + (3 - k) = 0$
A tangent touches once: discriminant = 0
$(2)^2 - 4(3 - k) = 0$
Answer: k = 2