Day 284 **A Level Maths: C Tier 10-a-day: Model Solutions**

1 Transforming a point on a graph (AQA, Edexcel, OCR, MEI, Eduqas, CIE)

The point P(−2, 4) lies on the curve y = f(x).
Write down the coordinates of the image of P on
(a) y = f(x + 2) (b) y = −f(x)

(a) y = f(x + 2) subtracts 2 from x, so P maps to (−4, 4).
(b) y = −f(x) changes the sign of y, so P maps to (−2, −4).
Answer: (a) (−4, 4) (b) (−2, −4)

2 Trapezium rule: strips shown (diagram) (AQA, Edexcel, OCR, MEI, Eduqas)

The diagram shows the curve $y = \sqrt{x + 2}$ from x = 0 to x = 3 divided into 4 strips of equal width.
Use the trapezium rule with these strips to estimate the shaded area to 3 s.f.

h = 0.75; y values: 1.414, 1.658, 1.871, 2.062, 2.236

Area $\approx \frac{0.75}{2}[1.414 + 2.236 + 2(5.591)]$

= 5.56
Answer: 5.56

3 Vertical motion under gravity: thrown upwards (AQA, Edexcel, OCR, MEI, Eduqas, CIE)

A stone is thrown vertically upwards with speed 19.6 m s^{-1}. Take g = 9.8 m s^{-2}.
(a) Find the time taken to reach its greatest height.
(b) Find the greatest height reached.

(a) v = u + at: 0 = 19.6 − 9.8t, so t = 2
(b) $v^2 = u^2 + 2as$: $0 = 19.6^2 - 2 \times 9.8 \times s$
$s = \frac{384.16}{19.6}$
Answer: (a) 2 s (b) 19.6 m

4 Modulus graphs (AQA, Edexcel, OCR, MEI, Eduqas, CIE)

Sketch $y = |x^2 - 9|$. Write down the coordinates where the graph meets the axes and the coordinates of the local maximum point.

y = 0: $x^2 = 9$, so x = ±3
x = 0: y = |−9| = 9. The part of $x^2 - 9$ below the x-axis is reflected in it.
The local maximum is at x = 0.
Answer: (3, 0), (−3, 0), (0, 9) ; maximum (0, 9)

5 Normal distribution: inverse (AQA, Edexcel, OCR, MEI, Eduqas, CIE)

$X \sim N(40, 8^2)$. Given that P(X > a) = 0.05, find a to 3 s.f.

P(Z > z) = 0.05, so z = 1.645 (from the inverse normal function)
a = 40 + 1.645 × 8
Answer: 53.2

6 Moments: reactions on a uniform beam (AQA, Edexcel, OCR, MEI)

A uniform beam AB of mass 30 kg and length 6 m rests horizontally on supports at A and B. A load of mass 10 kg is placed on the beam 4.5 m from A. Take g = 9.8 m s^{-2}. Find the reaction at B.

Moments about A:
$R_B \times 6 = 30 \times 9.8 \times 3 + 10 \times 9.8 \times 4.5$
$R_B \times 6 = 1323$
Answer: 220.5 N

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7 Hypothesis test for correlation (AQA, Edexcel, OCR, MEI, Eduqas)

12 pairs of data on height and arm span have $r = -0.65$.
Test, at the 5% significance level, for evidence of negative correlation. The critical value is -0.4973.

$H_0: \rho = 0$, $H_1: \rho < 0$
$r = -0.65$ is beyond the critical value -0.4973
Reject H_0
Answer: Reject H_0: there is evidence of negative correlation

8 Newton-Raphson: one or two steps (AQA, Edexcel, OCR, MEI, Eduqas)

$f(x) = x^2 - 6$.
Use the Newton-Raphson method with $x_0 = 2$ to find x_1 and x_2,
giving your answer to 3 decimal places.

$f'(x) = 2x$
$x_{n+1} = x_n - \dfrac{f(x_n)}{f'(x_n)}$
$x_1 = 2 - \dfrac{-2.000}{4.000} = 2.5000$
$x_2 = 2.5000 - \dfrac{0.2500}{5.000} = 2.4500$
Answer: $x_1 = 2.500$, $x_2 = 2.450$

9 Tangent condition for a line and a curve (AQA, Edexcel, OCR, MEI, Eduqas, CIE)

The line $y = -2x + k$ is a tangent to the curve
$y = x^2 - 6x$.
Find the value of k.

$x^2 - 6x = -2x + k$
$x^2 - 4x + (0 - k) = 0$
A tangent touches once: discriminant = 0
$(-4)^2 - 4(0 - k) = 0$
Answer: $k = -4$

10 Maximum value from R form (All boards)

$f(x) = 24 \sin x + 7 \cos x + 5$.
Write f(x) in the form $R \sin(x + \alpha) + 5$, with α in degrees to 1 d.p., and state the maximum value of f(x).

$R = \sqrt{24^2 + 7^2} = 25$, $\tan \alpha = \dfrac{7}{24}$ so $\alpha = 16.3°$
$f(x) = 25 \sin(x + 16.3°) + 5$
$\sin(x + \alpha)$ has maximum value 1, so f(max) = 25 + 5
Answer: f(x) = 25 sin(x + 16.3°) + 5; maximum 30