Day 283 **A Level Maths: C Tier 10-a-day: Model Solutions**

1 Finding a cubic from its graph (AQA, Edexcel, OCR, MEI, Eduqas, CIE)

The diagram shows the curve y = f(x), where f(x) is a cubic. Find f(x) in the form k(x − a)(x − b)(x − c).

The curve crosses the x-axis at x = 1, 3 and 4.
f(x) = k(x − 1)(x − 3)(x − 4)
At (0, −12): k × (−1) × (−3) × (−4) = −12
k = 1
Answer: f(x) = (x − 1)(x − 3)(x − 4)

2 Inverse trig functions (Edexcel, OCR)

Find the exact value, in radians, of
(a) arctan(−1) (b) arctan(1)

(a) y = arctan(−1) means tan y = −1 with $-\frac{\pi}{2} < y < \frac{\pi}{2}$: $y = -\frac{\pi}{4}$

(b) y = arctan(1) means tan y = 1 with $-\frac{\pi}{2} < y < \frac{\pi}{2}$: $y = \frac{\pi}{4}$

Answer: (a) $-\frac{\pi}{4}$ (b) $\frac{\pi}{4}$

3 Braking: deceleration and time (AQA, Edexcel, OCR, MEI, Eduqas, CIE)

A train travelling at 30 m s^{-1} brakes uniformly and comes to rest after travelling 150 m.
(a) Find the deceleration.
(b) Find the time taken to stop.

(a) $v^2 = u^2 + 2as$: $0 = 30^2 + 2 \times a \times 150$
so a = −3, a deceleration of 3

(b) $s = \frac{(u + v)}{2}$ t: 150 = 15 t

Answer: (a) 3 m s^{-2} (b) 10 s

4 Integrate (ax + b)^n and related (All boards)

Find ∫ cos(4x + 5) dx.

$\int \cos(ax + b)\,dx = \frac{1}{a}\sin(ax + b)$

Here a = 4

Answer: $\frac{1}{4}\sin(4x + 5) + c$

5 Solving equations with indices (AQA, Edexcel, OCR, MEI, Eduqas, CIE)

Solve $16^x = 512$.

$16 = 2^4$ and $512 = 2^9$
$2^{4x} = 2^9$, so 4x = 9

Answer: x = $\frac{9}{4}$

6 Binomial: at least one success (AQA, Edexcel, OCR, MEI, Eduqas, CIE)

Each customer buys a coffee with probability 0.3.
Find the probability that at least one of 4 customers buys a coffee, to 3 s.f.

P(zero) = 0.7^4
P(at least one) = $1 - 0.7^4$
Answer: 0.760

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7 Friction: constant speed, find the coefficient (All boards)

A trunk of mass 10 kg is pulled along rough horizontal ground at constant speed by a horizontal force of 49 N. Take g = 9.8 m s^{-2}. Find the coefficient of friction between the trunk and ground.

R = 10 × 9.8 = 98
Constant speed: F = P = 49
F = μR: 49 = μ × 98
Answer: 0.5

8 Domain and range (AQA, Edexcel, OCR, MEI, Eduqas, CIE)

f(x) = $\sqrt{x - 7} - 2$.
State the largest possible domain and the range of f.

$x - 7 \geq 0$, so $x \geq 7$.
$\sqrt{\ldots} \geq 0$, so $f(x) \geq -2$.
Answer: Domain $x \geq 7$, range $f(x) \geq -2$

9 Conditional probability (Bayes) (AQA, Edexcel, OCR, MEI, Eduqas, CIE)

The probability that a patient takes the drug is 0.2. If so, the probability that the patient recovers is 0.5; otherwise it is 0.3. Given that the patient recovers, find the probability that a patient takes the drug.

P(both) = 0.2 × 0.5 = 0.1; P(not first and second) = 0.8 × 0.3 = 0.24
P(second) = 0.1 + 0.24 = 0.34
P = 0.1 ÷ 0.34
Answer: $\frac{5}{17}$ = 0.294

10 Estimate with a small angle (AQA, Edexcel, OCR, MEI, Eduqas)

Use $\sin\theta \approx \theta$ to estimate sin 3°, giving your answer to 3 significant figures.

Convert to radians: 3° = $\frac{3\pi}{180}$ = 0.05236 rad
sin 3° ≈ 0.0524
Answer: 0.0524