Day 281 **A Level Maths: C Tier 10-a-day: Model Solutions**

1 Linear inequalities (AQA, Edexcel, OCR, MEI, Eduqas, CIE)

Solve $-17 \leq 3x - 5 < -2$.

Subtract -5 from each part: $-12 \leq 3x < 3$
Divide each part by 3.
Answer: $-4 \leq x < 1$

2 Stationary points of a cubic (All boards)

$y = x^3 - 6x^2 + 9x + 3$.
Find the coordinates of the stationary points and determine their nature.

$dy/dx = 3x^2 - 12x + 9 = 3(x - 1)(x - 3) = 0$
$x = 1$: $y = 7$; $x = 3$: $y = 3$
$d^2y/dx^2 = 6x - 12$: at $x = 1$ it is -6 (maximum), at $x = 3$ it is 6 (minimum)
Answer: (1, 7) maximum; (3, 3) minimum

3 Velocity zero (quadratic in t) (AQA, Edexcel, OCR, MEI, Eduqas, CIE)

A particle moves in a straight line. Its velocity at time t seconds is $v = (t^2 - 4t + 3)$ m s^{-1}.
(a) Find the times when the particle is at rest.
(b) Find its acceleration at the first of these times.

(a) $t^2 - 4t + 3 = 0$
$(t - 1)(t - 3) = 0$
(b) $a = \frac{dv}{dt} = 2t - 4$; at $t = 1$, $a = 2 - 4$
Answer: (a) t = 1 and t = 3 (b) -2 m s^{-2}

4 Continuous random variable: probability and mean (CIE)

X has probability density function $f(x) = \frac{3}{64}x^2$ for $0 \leq x \leq 4$, and 0 otherwise.
Find $P(X < 1)$.

$P(X < 1) = \int$ from 0 to 1 of $\frac{3}{64}x^2\,dx$
$= [\frac{3}{192}x^3]$ from 0 to 1
Answer: $\frac{1}{64}$

5 Projectile launched with velocity components (AQA, Edexcel, OCR, MEI)

A stone is projected from level ground with velocity $(8\mathbf{i} + 39.2\mathbf{j})$ m s^{-1} ($\mathbf{i}$ horizontal, $\mathbf{j}$ vertically up). Take g = 9.8 m s^{-2}.
(a) Find the time of flight.
(b) Find the horizontal range.

(a) Vertically: $0 = 39.2\,t - 4.9t^2$
$t = \frac{39.2}{4.9} = 8$
(b) Range = 8×8
Answer: (a) 8 s (b) 64 m

6 Gradient of a tangent to a circle (AQA, Edexcel, OCR, MEI, Eduqas, CIE)

The circle $(x - 3)^2 + (y + 3)^2 = 25$ passes through P(7, 0).
Find the gradient of the tangent to the circle at P.

Centre C(3, -3).
Gradient of CP = $\frac{0 - (-3)}{7 - 3} = \frac{3}{4}$
The tangent is perpendicular to CP: gradient = $-1 \div \frac{3}{4}$
Answer: $-\frac{4}{3}$

Day 281 **A Level Maths: C Tier 10-a-day: Model Solutions**

7 Vector geometry with a diagram (OAB) (All boards)

In the diagram, OA = **a**, OB = **b**, and P lies on AB with AP : PB = 2 : 1.
Find AB and AP in terms of **a** and **b**.

AB = OB − OA = $\mathbf{b} - \mathbf{a}$

AP = $\frac{2}{3}$ AB = $\frac{2}{3}(\mathbf{b} - \mathbf{a})$

Answer: AB = $\mathbf{b} - \mathbf{a}$; AP = $\frac{2}{3}(\mathbf{b} - \mathbf{a})$

8 Solving equations with indices (AQA, Edexcel, OCR, MEI, Eduqas, CIE)

Solve $9^x = 59049$.

$9 = 3^2$ and $59049 = 3^{10}$

$3^{2x} = 3^{10}$, so $2x = 10$

Answer: x = 5

9 Sector: arc length and area (diagram) (All boards)

The diagram shows sector OAB of a circle.
Find, in terms of π, (a) the arc length AB, (b) the area of the sector.

(a) Arc length = $r\theta = 12 \times \frac{\pi}{3} = 4\pi$ cm

(b) Area = $\frac{1}{2}r^2\theta = \frac{1}{2} \times 12^2 \times \frac{\pi}{3} = 24\pi$ cm^2

Answer: (a) 4π cm (b) 24π cm^2

10 Conditional probability (Bayes) (AQA, Edexcel, OCR, MEI, Eduqas, CIE)

The probability that a patient takes the drug is 0.3. If so, the probability that the patient recovers is 0.7; otherwise it is 0.1.
Given that the patient recovers, find the probability that a patient takes the drug.

P(both) = 0.3 × 0.7 = 0.21; P(not first and second) = 0.7 × 0.1 = 0.07

P(second) = 0.21 + 0.07 = 0.28

P = 0.21 ÷ 0.28

Answer: $\frac{3}{4}$ = 0.75