Day 28 **A Level Maths: C Tier 10-a-day: Model Solutions**

1 Exact trigonometric values (All boards)

Write down the exact value of tan 45°.

Use the special angle values: tan 45° = 1

Answer: 1

2 Arithmetic sequences: nth term (AQA, Edexcel, OCR, MEI, Eduqas, CIE)

In an arithmetic sequence $u_4 = 12$ and $u_{11} = 54$.

Find the first term and the common difference.

Subtract: $(11 - 4)d = 54 - 12$

$7d = 42$, so $d = 6$

$a = u_4 - 3d = 12 - 18 = -6$

Answer: $a = -6$, $d = 6$

3 Integrating velocity to find displacement (AQA, Edexcel, OCR, MEI, Eduqas, CIE)

A particle moves in a straight line with velocity $v = (3t^2 + 6t + 5)$ m s^{-1} at time t seconds.

Find the distance travelled between t = 1 and t = 3 (the particle does not turn round).

$s = \int v\,dt = t^3 + 3t^2 + 5t$

Distance = $s(3) - s(1)$

$= 69 - 9$

Answer: 60 m

4 Equation of a transformed curve (AQA, Edexcel, OCR, MEI, Eduqas, CIE)

The curve $y = x^2 - x + 1$ is translated by the vector $\begin{pmatrix}1\\5\end{pmatrix}$.

Find the equation of the new curve, giving your answer in the form $y = ax^2 + bx + c$.

Replace x by $(x - 1)$ and add 5:

$y = f(x - 1) + 5$

Expand and collect terms.

Answer: $y = x^2 - 3x + 8$

5 Continuous random variable: finding k (CIE)

The continuous random variable X has probability density function $f(x) = k(x + 3)$ for $0 \le x \le 2$, and 0 otherwise.

Find the value of k.

$\int$ from 0 to 2 of $k(x + 3)\,dx = 1$

$k[\frac{x^2}{2} + 3x]$ from 0 to 2 $= k(8) = 1$

Answer: $k = \frac{1}{8}$

6 Particle on a smooth table connected to a hanging particle (All boards)

A 4 kg particle on a smooth horizontal table is joined by a light string over a smooth pulley to a hanging 4 kg particle, as shown. The system is released from rest. Take g = 9.8 m s^{-2}. Find the tension.

Table particle: $T = 4a$

Hanging particle: $4 \times 9.8 - T = 4a$

Adding: $39.2 = 8a$

Answer: 19.6 N

7 Partial fractions (Edexcel, OCR, MEI, CIE, AQA, Eduqas)

Express $\frac{-2x-1}{(x+2)(x+1)}$ in partial fractions.

Write $\frac{-2x-1}{(x+2)(x+1)} = \frac{A}{(x+2)} + \frac{B}{(x+1)}$

$-2x - 1 = A(x + 1) + B(x + 2)$

$x = -2$: $3 = A(-1)$ so $A = -3$

$x = -1$: $1 = B(1)$ so $B = 1$

Answer: $\mathbf{\frac{-3}{(x+2)} + \frac{1}{(x+1)}}$

8 Integrate using partial fractions (All boards)

$\frac{x}{(x+2)(x+3)} \equiv \frac{A}{x+2} + \frac{B}{x+3}$.

Find A and B, and hence find $\int \frac{x}{(x+2)(x+3)}\,dx$.

$x \equiv A(x + 3) + B(x + 2)$

$x = -2$ gives $A = -2$; $x = -3$ gives $B = 3$

$\int \left(\frac{-2}{x+2} + \frac{3}{x+3}\right) dx = -2 \ln|x + 2| + 3 \ln|x + 3| + c$

Answer: $A = -2$, $B = 3$; $-2 \ln|x + 2| + 3 \ln|x + 3| + c$

9 Sampling methods (AQA, Edexcel, OCR, MEI, Eduqas)

A reporter asks the first 15 people who walk out of a college for their views.
Name the sampling method used.

This is opportunity sampling: people are chosen only because they are available at the time.

Answer: opportunity sampling

10 Differentiate e^x, ln x, sin and cos (All boards)

Differentiate with respect to x:

(a) $e^{4x} + \sin 2x$

(b) $4\,e^{2x}$

(a) $dy/dx = 4\,e^{4x} + 2\cos 2x$

(b) $dy/dx = 8\,e^{2x}$

Answer: (a) $4\,e^{4x} + 2\cos 2x$ (b) $8\,e^{2x}$