Day 278 **A Level Maths: C Tier 10-a-day: Model Solutions**

1 Differentiate e^x, ln x, sin and cos (All boards)

Differentiate with respect to x:

(a) $5\,e^{4x}$

(b) $5 \cos 4x$

(a) $dy/dx = 20\,e^{4x}$

(b) $dy/dx = -20 \sin 4x$

Answer: (a) $20\,e^{4x}$ (b) $-20 \sin 4x$

2 Differentiating velocity to find acceleration (AQA, Edexcel, OCR, MEI, Eduqas, CIE)

A particle moves in a straight line. Its velocity at time t seconds is v m s^{-1}, where $v = t^2 - 6t + 1$.

(a) Find its acceleration when t = 2.

(b) Find the time when its acceleration is zero.

$a = \frac{dv}{dt} = 2t - 6$

(a) $t = 2$: $a = 2 \times 2 - 6$

(b) $2t - 6 = 0$, so $t = \frac{6}{2}$

Answer: (a) -2 m s^{-2} (b) 3 s

3 Addition formulae: exact values (All boards)

By writing 75° as 30° + 45°, find the exact value of sin 75°.

$\sin(30° + 45°) = \sin 30° \cos 45° + \cos 30° \sin 45°$

$= \frac{1}{2} \times \frac{\sqrt{2}}{2} + \frac{\sqrt{3}}{2} \times \frac{\sqrt{2}}{2}$

Simplify: $\frac{\sqrt{6} + \sqrt{2}}{4}$

Answer: $\frac{\sqrt{6} + \sqrt{2}}{4}$

4 Binomial: cumulative probabilities (AQA, Edexcel, OCR, MEI, Eduqas, CIE)

$X \sim B(10, 0.1)$. Find $P(1 \leq X \leq 2)$, giving your answer to 3 s.f.

$P(1 \leq X \leq 2) = P(X \leq 2) - P(X \leq 0)$

$= 0.9298 - 0.3487$

Answer: 0.581

5 Modulus equations (AQA, Edexcel, OCR, MEI, Eduqas, CIE)

Solve $|3x + 3| = 12$.

$3x + 3 = 12$ or $3x + 3 = -12$

$x = 3$ or $x = -5$

Answer: x = 3 or x = −5

6 Maximum speed from calculus (AQA, Edexcel, OCR, MEI, Eduqas, CIE)

A particle moves in a straight line. Its velocity at time t seconds ($0 \leq t \leq 2$) is v = $(-t^2 + 2t)$ m s^{-1}. Find the maximum velocity and the time at which it occurs.

$a = \frac{dv}{dt} = 2 - 2t$

Maximum when $a = 0$: $t = \frac{2}{2} = 1$

$v = 2 \times 1 - 1 \times 1^2$

Answer: 1 m s^{-1} at t = 1

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7 Simplifying algebraic indices (AQA, Edexcel, OCR, MEI, Eduqas, CIE)

Simplify $(4x^4)^3 \div 16x^{-1}$.

$(4x^4)^3 = 64x^{12}$

$64x^{12} \div 16x^{-1}$: divide the numbers and subtract the powers

$= 4x^{13}$

Answer: $4x^{13}$

8 Conditional probability (Bayes) (AQA, Edexcel, OCR, MEI, Eduqas, CIE)

The probability that a patient takes the drug is 0.3. If so, the probability that the patient recovers is 0.8; otherwise it is 0.1. Given that the patient recovers, find the probability that a patient takes the drug.

P(both) = 0.3 × 0.8 = 0.24; P(not first and second) = 0.7 × 0.1 = 0.07

P(second) = 0.24 + 0.07 = 0.31

P = 0.24 ÷ 0.31

Answer: $\frac{24}{31}$ = 0.774

9 Newton-Raphson: one or two steps (AQA, Edexcel, OCR, MEI, Eduqas)

$f(x) = \ln x + x - 2$.

Use the Newton-Raphson method with $x_0 = 3$ to find x_1 and x_2,

giving your answer to 3 decimal places.

$f'(x) = \frac{1}{x} + 1$

$x_{n+1} = x_n - \frac{f(x_n)}{f'(x_n)}$

$x_1 = 3 - \frac{2.099}{1.333} = 1.4260$

$x_2 = 1.4260 - \frac{-0.2191}{1.701} = 1.5548$

Answer: $x_1 = 1.426$, $x_2 = 1.555$

10 Parametric equations to Cartesian form (Edexcel, OCR, MEI, CIE, AQA, Eduqas)

A curve has parametric equations

$x = t - 3$, $y = 3t^2 + 5$.

Find a Cartesian equation of the curve.

From $x = t - 3$: $t = x + 3$

Substitute into y: $y = 3(x + 3)^2 + 5$

Answer: $y = 3(x + 3)^2 + 5$