Day 276 **A Level Maths: C Tier 10-a-day: Model Solutions**

1 Differentiating displacement to find velocity (AQA, Edexcel, OCR, MEI, Eduqas, CIE)

A particle moves in a straight line. Its displacement from O at time t seconds is s metres, where $s = 3t^3 - 3t^2 + t$.
Find its velocity when t = 2.

$v = \frac{ds}{dt} = 9t^2 - 6t + 1$

When t = 2: $v = 9 \times 2^2 + -6 \times 2 + 1$

Answer: 25 m s^{-1}

2 2D vectors: magnitude and direction (All boards)

The vector $\mathbf{p} = -15\mathbf{i} - 8\mathbf{j}$.
Find (a) |p|, (b) the angle p makes with the positive x-axis, measured anticlockwise (1 d.p.).

(a) $|\mathbf{p}| = \sqrt{15^2 + 8^2} = 17$

(b) Acute angle = $\tan^{-1}(\frac{8}{15}) = 28.1°$

p is in the third quadrant, so the angle is 208.1°

Answer: (a) 17 (b) 208.1°

3 Describing transformations of graphs (AQA, Edexcel, OCR, MEI, Eduqas, CIE)

Describe the single transformation that maps the graph of y = f(x) onto the graph of $y = f(\frac{x}{2})$.

$y = f(\frac{x}{2})$ is obtained from y = f(x) by a:

stretch parallel to the x-axis, scale factor 2

Answer: stretch parallel to the x-axis, scale factor 2

4 Double angle: find sin 2A and cos 2A (All boards)

A is acute and $\sin A = \frac{8}{17}$.

Find the exact values of sin 2A and cos 2A.

$\cos A = \sqrt{1 - \sin^2 A} = \frac{15}{17}$

$\sin 2A = 2 \sin A \cos A = 2 \times \frac{8}{17} \times \frac{15}{17} = \frac{240}{289}$

$\cos 2A = \cos^2 A - \sin^2 A = \frac{225}{289} - \frac{64}{289} = \frac{161}{289}$

Answer: sin 2A = $\frac{240}{289}$, cos 2A = $\frac{161}{289}$

5 Quadratic inequalities (AQA, Edexcel, OCR, MEI, Eduqas, CIE)

Solve x + 3 > 18.

Solve $x^2 + 3x - 18 = 0$: $(x + 6)(x - 3) = 0$
Critical values x = −6 and x = 3.
The graph is a ∪-shaped parabola, so take the regions outside the roots.
Answer: x < −6 or x > 3

6 Tension in a string supporting a hanging mass (All boards)

A particle of mass 8 kg hangs from a light string and is raised at constant speed. Taking g = 9.8 m s^{-2}, find the tension in the string.

No acceleration, so T = mg
T = 8 × 9.8
Answer: 78.4 N

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7 Discrete random variable: E(X) and Var(X) (AQA, Edexcel, OCR, MEI, Eduqas, CIE)

The discrete random variable X has this distribution.
x: 0, 1, 4, 5
P(X = x): 0.1, 0.1, 0.3, 0.5
Find E(X) and Var(X).

$E(X) = \Sigma xp = 0\times0.1 + 1\times0.1 + 4\times0.3 + 5\times0.5 = 3.8$

$E(X^2) = \Sigma x^2 p = 17.4$

$\text{Var}(X) = 17.4 - 3.8^2$

Answer: E(X) = 3.8, Var(X) = 2.96

8 Integrate using partial fractions (All boards)

$\frac{x+4}{(x+2)(x+3)} \equiv \frac{A}{x+2} + \frac{B}{x+3}$.

Find A and B, and hence find $\int \frac{x+4}{(x+2)(x+3)}\,dx$.

$x + 4 \equiv A(x + 3) + B(x + 2)$

$x = -2$ gives $A = 2$; $x = -3$ gives $B = -1$

$\int \left(\frac{2}{x+2} + \frac{-1}{x+3}\right) dx = 2\ln|x + 2| + -1\ln|x + 3| + c$

Answer: A = 2, B = −1; $2\ln|x + 2| - \ln|x + 3| + c$

9 Conditional probability (Bayes) (AQA, Edexcel, OCR, MEI, Eduqas, CIE)

The probability that it rains is 0.2. If so, the probability that Sam is late is 0.7; otherwise it is 0.2.
Given that Sam is late, find the probability that it rains.

P(both) = 0.2 × 0.7 = 0.14; P(not first and second) = 0.8 × 0.2 = 0.16

P(second) = 0.14 + 0.16 = 0.3

P = 0.14 ÷ 0.3

Answer: $\frac{7}{15}$ = 0.467

10 Equation of a reciprocal graph from asymptotes (AQA, Edexcel, OCR, MEI, Eduqas, CIE)

The diagram shows the graph of $y = \frac{k}{x-a} + c$ with its asymptotes. Find the values of a, c and k.

The asymptotes are $x = 2$ and $y = -2$, so $a = 2$ and $c = -2$.

Through $(0, -3)$: $-3 = \frac{k}{0-2} + -2$

$\frac{k}{-2} = -1$, so $k = 2$

Answer: a = 2, c = −2, k = 2