Day 273 **A Level Maths: C Tier 10-a-day: Model Solutions**

1 Arithmetic sequences: nth term (AQA, Edexcel, OCR, MEI, Eduqas, CIE)

In an arithmetic sequence $u_5 = 12$ and $u_8 = 24$.

Find the first term and the common difference.

Subtract: $(8 - 5)d = 24 - 12$

$3d = 12$, so $d = 4$

$a = u_5 - 4\,d = 12 - 16 = -4$

Answer: $a = -4$, $d = 4$

2 Integrate standard functions (All boards)

Find

(a) $\int 8\sec^2 4x\,dx$

(b) $\int 8\cos 4x\,dx$

Use the standard integrals and divide by the coefficient of x where needed.

(a) $= 2\tan 4x + c$

(b) $= 2\sin 4x + c$

Answer: (a) $2\tan 4x + c$ (b) $2\sin 4x + c$

3 Outliers (Edexcel, OCR, MEI)

For a data set $Q_1 = 28$ and $Q_3 = 46$.

A value is an outlier if it is more than 1.5 × IQR above Q_3 or below Q_1.

Determine whether -1 and 38 are outliers.

IQR $= 46 - 28 = 18$, so $1.5 \times \text{IQR} = 27$

Lower limit $= 28 - 27 = 1$; upper limit $= 46 + 27 = 73$

-1 is an outlier; 38 is not an outlier

Answer: -1 is an outlier; 38 is not an outlier

4 Domain and range (AQA, Edexcel, OCR, MEI, Eduqas, CIE)

$f(x) = (x + 1)^2 + 2$, $x \geq 0$.

Find the range of f.

For $x \geq 0$ the least value is at $x = 0$ (the vertex $x = -1$ is not allowed).

$f(0) = 1 + 2 = 3$, and f increases for $x > 0$.

Answer: $f(x) \geq 3$

5 Solve with a shifted angle (All boards)

Solve $\sin(x - 15°) = \frac{1}{2}$ for $0° \leq x \leq 360°$.

Let $u = x - 15°$, so $-15° \leq u \leq 345°$

$\sin u = \frac{1}{2}$: $u = 30°, 150°$

$x = u + 15°$: $x = 45°, 165°$

Answer: $x = 45°, 165°$

6 Completing the square (AQA, Edexcel, OCR, MEI, Eduqas, CIE)

Write $x^2 - 4x + 2$ in the form $(x + p)^2 + q$, and state the coordinates of the minimum point of $y = x^2 - 4x + 2$.

$x^2 - 4x = (x - 2)^2 - 4$

$x^2 - 4x + 2 = (x - 2)^2 - 2$

The minimum is where $(x - 2)^2 = 0$, i.e. $x = 2$

Answer: $(x - 2)^2 - 2$; minimum $(2, -2)$

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7 Resultant of perpendicular forces (All boards)

Two forces of magnitudes 8 N and 15 N act on a particle at right angles to each other. Find (a) the magnitude of the resultant, (b) the angle it makes with the 8 N force, to 1 d.p.

(a) $R^2 = 8^2 + 15^2 = 289$

$R = \sqrt{289}$

(b) $\tan\theta = \frac{15}{8}$

Answer: (a) 17 N (b) 61.9°

8 Two-tailed binomial test (AQA, Edexcel, OCR, MEI, Eduqas, CIE)

$X \sim B(25, p)$. H_0: $p = 0.3$, H_1: $p \neq 0.3$. In a sample, $x = 4$.

Test at the 10% significance level.

Two-tailed test, so compare with $0.1 \div 2 = 0.05$

$P(X \leq 4) = 0.0905$

$0.0905 > 0.05$, so do not reject H_0

Answer: Do not reject H_0 ($P(X \leq 4) = 0.0905$)

9 Solve a^x = b using logarithms (All boards)

Solve $7^{x+2} = 100$.

Give your answer to 3 significant figures.

Take logs: $(x + 2) \log 7 = \log 100$

$x + 2 = 2.367$, so $x = 0.367$

Answer: $x = 0.367$

10 Power of an engine: driving force (CIE)

A train's engine works at a constant rate of 12 kW. Find the driving force when the vehicle is moving at 12 m s^{-1}.

$P = Fv$

$12000 = F \times 12$

Answer: 1000 N