Day 272 **A Level Maths: C Tier 10-a-day: Model Solutions**

1 Mean from a frequency table (AQA, Edexcel, OCR, MEI, Eduqas, CIE)

The table summarises the number of goals scored in a game by 20 people.
Goals, x: 0, 1, 2, 3, 4
Frequency, f: 6, 3, 6, 2, 3
Find the mean.

$\Sigma fx = 6\times0 + 3\times1 + 6\times2 + 2\times3 + 3\times4 = 33$
Mean = $\Sigma fx \div \Sigma f = 33 \div 20$
Answer: 1.65

2 Solve a^x = b using logarithms (All boards)

Solve $1.5^{3x} = 30$.
Give your answer to 3 significant figures.

Take logs: $3x \log 1.5 = \log 30$
$x = \dfrac{\log 30}{3 \log 1.5} = 2.80$
Answer: x = 2.80

3 Regression: prediction and reliability (AQA, Edexcel, OCR, MEI, Eduqas)

For 7 data pairs, with advertising spend (£s hundreds) from 4 to 14, the regression line is S = 19 + 7a.
Estimate sales (£s thousands) when a = 20 and comment on the reliability.

$S = 19 + 7 \times 20 = 159$
20 is outside the range 4 to 14, so this is extrapolation.
Answer: S = 159; estimate is unreliable, as 20 is outside the data range (extrapolation)

4 Recurrence relations (AQA, Edexcel, OCR, MEI, Eduqas, CIE)

A sequence is defined by $u_{n+1} = -2u_n - 5$, $u_1 = 2$.
Find u_2, u_3 and u_4.

$u_2 = (-2) \times 2 - 5 = -9$
$u_3 = (-2) \times (-9) - 5 = 13$
$u_4 = (-2) \times 13 - 5 = -31$
Answer: $u_2 = -9$, $u_3 = 13$, $u_4 = -31$

5 Stone thrown down from a cliff (AQA, Edexcel, OCR, MEI, Eduqas, CIE)

A stone is thrown vertically downwards at 4 m s^{-1} from the top of a cliff and hits the sea after 4 s. Taking g = 9.8 m s^{-2}, find the height of the cliff.

$s = ut + \frac{1}{2}at^2$
$s = 4 \times 4 + \frac{1}{2} \times 9.8 \times 4^2$
$s = 16 + 78.4$
Answer: 94.4 m

6 Separable differential equations (All boards)

Given that dy/dx = 4xy and y = 5 when x = 0,
find y in terms of x.

$\frac{1}{y}\,dy = 4x\,dx$, so $\ln y = 2x^2 + c$
x = 0, y = 5: $c = \ln 5$
$y = 5\,e^{2x^2}$
Answer: $y = 5\,e^{2x^2}$

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7 Linear and quadratic simultaneous equations (AQA, Edexcel, OCR, MEI, Eduqas, CIE)

Solve the simultaneous equations:

$y = x^2 - 4x - 18$, $y = -3x + 2$.

$x^2 - 4x - 18 = -3x + 2$

$x^2 - x - 20 = 0$

$(x + 4)(x - 5) = 0$

Use $y = -3x + 2$ to find each y.

Answer: $x = -4$, $y = 14$ and $x = 5$, $y = -13$

8 Integrate using partial fractions (All boards)

$\frac{x - 2}{(x + 4)(x + 1)} \equiv \frac{A}{x + 4} + \frac{B}{x + 1}$.

Find A and B, and hence find $\int \frac{x - 2}{(x + 4)(x + 1)}\, dx$.

$x - 2 \equiv A(x + 1) + B(x + 4)$

$x = -4$ gives $A = 2$; $x = -1$ gives $B = -1$

$\int \left(\frac{2}{x + 4} + \frac{-1}{x + 1}\right) dx = 2 \ln|x + 4| + -1 \ln|x + 1| + c$

Answer: $A = 2$, $B = -1$; $2 \ln|x + 4| - \ln|x + 1| + c$

9 Binomial expansion: non-integer powers (Edexcel, AQA, OCR, MEI, Eduqas, CIE)

Expand $(1 - x)^{\frac{1}{3}}$ in ascending powers of x up to and including the term in x^2. State the range of values of x for which the expansion is valid.

$(1 + y)^n = 1 + ny + \frac{n(n - 1)}{2!}y^2 + \ldots$ with $n = \frac{1}{3}$, $y = -x$

$= 1 + \frac{1}{3}(-x) + \frac{\frac{1}{3}\left(\left(-\frac{2}{3}\right)\right)}{2}(-x)^2$

$= 1 - \frac{1}{3}x - \frac{1}{9}x^2$

Valid when $|-x| < 1$, i.e. $|x| < 1$

Answer: $1 - \frac{1}{3}x - \frac{1}{9}x^2$; $|x| < 1$

10 Smooth inclined plane: acceleration and speed (All boards)

A particle of mass 2 kg slides from rest down a smooth plane inclined at angle α to the horizontal, where $\sin\alpha = \frac{3}{5}$. Take $g = 9.8$ m s^{-2}.

Find (a) its acceleration, (b) its speed after 3 s.

(a) Parallel to slope: $mg \sin\alpha = ma$

$a = 9.8 \times \frac{3}{5}$

(b) $v = u + at = 0 + 5.88 \times 3$

Answer: (a) 5.88 m s^{-2} (b) 17.64 m s^{-1}