Day 271 **A Level Maths: C Tier 10-a-day: Model Solutions**

1 Discrete random variable: finding k (AQA, Edexcel, OCR, MEI, Eduqas, CIE)

The discrete random variable X has $P(X = x) = k(x^2 + 2)$ for $x = 0, 1, 2, 3$.
(a) Find the value of k.
(b) Find $P(X \geq 1)$.

(a) Probabilities sum to 1: $k(2 + 3 + 6 + 11) = 1$

$k = \frac{1}{22}$

(b) $P(X \geq 1) = \frac{3}{22} + \frac{6}{22} + \frac{11}{22}$

Answer: (a) $k = \frac{1}{22}$ (b) $\frac{10}{11}$

2 Line through two axis intercepts (AQA, Edexcel, OCR, MEI, Eduqas, CIE)

The diagram shows a line meeting the axes at A and B.
Find (a) the equation of AB in the form $px + qy = r$ with integers p, q and r, (b) the length AB.

Gradient $= \frac{3 - 0}{0 - 2} = -\frac{3}{2}$

Using $y = mx + c$ with $c = 3$: $y = -\frac{3}{2}x + 3$, so $3x + 2y = 6$

$AB^2 = 2^2 + 3^2 = 13$

Answer: (a) $3x + 2y = 6$ (b) $\sqrt{13}$

3 Two-stage motion: acceleration then constant speed (AQA, Edexcel, OCR, MEI, Eduqas, CIE)

A train starts from rest and accelerates uniformly at 1 m s^{-2} for 6 s, then travels at constant speed for a further 20 s. Find the total distance travelled.

Speed reached: $v = 1 \times 6 = 6$ m s^{-1}

Stage 1: $s = \frac{1}{2} \times 6 \times 6 = 18$

Stage 2: $s = 6 \times 20 = 120$

Answer: 138 m

4 Hypothesis test for correlation (AQA, Edexcel, OCR, MEI, Eduqas)

8 pairs of data on height and arm span have $r = 0.47$.
Test, at the 5% significance level, for evidence of positive correlation. The critical value is 0.6215.

$H_0: \rho = 0$, $H_1: \rho > 0$

$r = 0.47$ is not beyond the critical value 0.6215

Do not reject H_0

Answer: Do not reject H_0: there is not enough evidence of positive correlation

5 Iteration $x_{n+1} = g(x_n)$ (All boards)

$x_{n+1} = \sqrt{5x_n + 2}$, with $x_0 = 2$.
Find x_1, x_2 and x_3 to 3 decimal places.

$x_1 = 3.4641$

$x_2 = 4.3955$

$x_3 = 4.8967$

Answer: $x_1 = 3.464$, $x_2 = 4.396$, $x_3 = 4.897$

6 Geometric series: sum of n terms (AQA, Edexcel, OCR, MEI, Eduqas, CIE)

Find the sum of the first 6 terms of the geometric series $4 + 8 + 16 + \ldots$

$a = 4$, $r = 2$, $n = 6$

$S_n = \frac{a(r^n - 1)}{r - 1} = \frac{4(2^6 - 1)}{2 - 1}$

$= 252$

Answer: 252

Day 271 **A Level Maths: C Tier 10-a-day: Model Solutions**

7 Solve with a shifted angle (All boards)

Solve $\sin(x - 20°) = \frac{\sqrt{2}}{2}$ for $0° \le x \le 360°$.

Let $u = x - 20°$, so $-20° \le u \le 340°$

$\sin u = \frac{\sqrt{2}}{2}$: $u = 45°, 135°$

$x = u + 20°$: $x = 65°, 155°$

Answer: $x = 65°, 155°$

8 Horizontal projection from a cliff (AQA, Edexcel, OCR, MEI)

A ball is thrown horizontally at 9 m s^{-1} from the top of a vertical cliff of height 44.1 m above the sea. Take $g = 9.8$ m s^{-2}. Find (a) the time taken to reach the sea, (b) the horizontal distance of the landing point from the cliff.

(a) Vertically: $u = 0$, $s = 44.1$, $a = 9.8$

$44.1 = \frac{1}{2} \times 9.8 \times t^2$, so $t^2 = 9$

(b) Horizontal distance = speed × time = 9×3

Answer: (a) 3 s (b) 27 m

9 Implicit differentiation (AQA, Edexcel, OCR, MEI, Eduqas, CIE)

A curve has equation $x^2 + y^2 = 25$.
Find dy/dx in terms of x and y, and hence the gradient at $(-3, 4)$.

Differentiate both sides with respect to x (using dy/dx for y terms)

$dy/dx = -\frac{x}{y}$

At $(-3, 4)$: $dy/dx = \frac{3}{4}$

Answer: $dy/dx = -\frac{x}{y}$; gradient $\frac{3}{4}$

10 Simplifying algebraic indices (AQA, Edexcel, OCR, MEI, Eduqas, CIE)

Simplify $(2x)^3 \div 2x^3$.

$(2x)^3 = 8x^3$

$8x^3 \div 2x^3$: divide the numbers and subtract the powers

$= 4$

Answer: 4